Boundary Layers – The Karman Momentum Integral Equation (KMIE)

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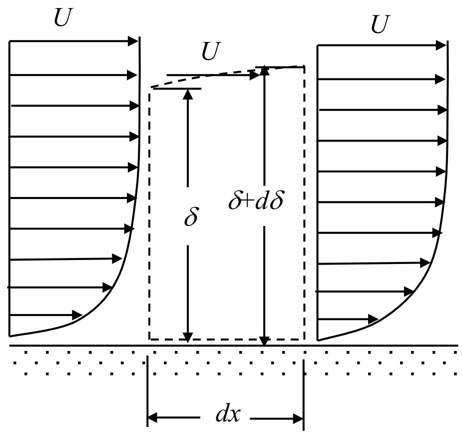

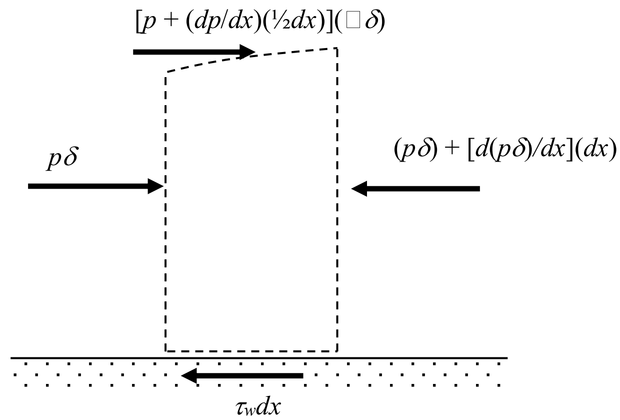

$$\underbrace{-\dot{m}_{\text{top}}}_{\text{in through top}} \underbrace{-\int_0^{\delta} \rho u dy}_{\text{in at left}} + \underbrace{\left[\int_0^{\delta} \rho u dy + \frac{d}{dx}\left(\int_0^{\delta} \rho u dy\right) dx\right]}_{\text{out through right}} = 0$$

$$\therefore \dot{m}_{\text{top}} = \frac{d}{dx}\left(\int_0^{\delta} \rho u dy\right) dx$$

$$F_{x,\text{on CV}} = \frac{d}{dt}\int_{\text{CV}} u_x \rho dV + \int_{\text{CS}} u_x \rho\left(\mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right)$$

$$F_{x,\text{on CV}} = \underbrace{p\delta}_{\text{left}} \underbrace{-\left[p\delta + \frac{d}{dx}(p\delta)(dx)\right]}_{\text{right}} + \underbrace{\left[p + \frac{dp}{dx}\left(\tfrac{1}{2} dx\right)\right](d\delta)}_{\text{top}} \underbrace{-\tau_w dx}_{\text{bottom}}$$

$$\int_{\text{CS}} u_x \rho\left(\mathbf{u}_{\text{rel}} \cdot d\mathbf{A}\right) = \underbrace{-\int_{y=0}^{y=\delta} \rho u^2 dy}_{\text{left}} + \underbrace{\left[\int_{y=0}^{y=\delta} \rho u^2 dy + \frac{d}{dx}\left(\int_{y=0}^{y=\delta} \rho u^2 dy\right) dx\right]}_{\text{right}} \underbrace{-U \frac{d}{dx}\left[\int_{y=0}^{y=\delta} \rho u dy\right] dx}_{\text{top}}$$

$$-\frac{dp}{dx}\delta dx - \tau_w dx = \frac{d}{dx}\left[\int_0^{\delta} \rho u^2 dy\right] dx - U \frac{d}{dx}\left[\int_0^{\delta} \rho u dy\right] dx$$

$$\Rightarrow -\frac{dp}{dx}\delta - \tau_w = \frac{d}{dx}\left[\int_0^{\delta} \rho u^2 dy\right] - U \frac{d}{dx}\left[\int_0^{\delta} \rho u dy\right]$$

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$$p + \tfrac{1}{2}\rho U^2 = \text{constant} \quad \Rightarrow \quad \frac{dp}{dx} + \rho U \frac{dU}{dx} = 0$$

$$\therefore \frac{dp}{dx} = -\rho U \frac{dU}{dx}$$

$$\boxed{\frac{\tau_w}{\rho} = \frac{d}{dx}\left[U^2 \delta_M\right] + \delta_D U \frac{dU}{dx}}$$