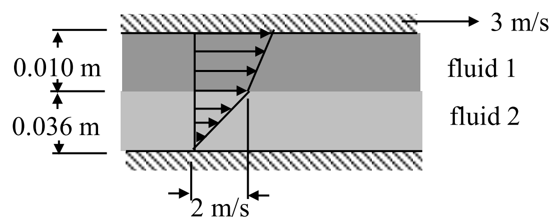
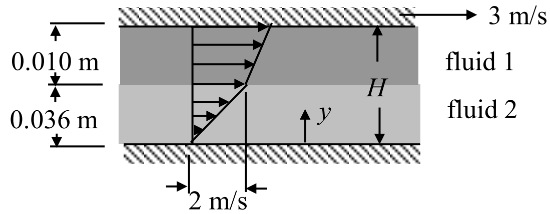
The no-slip condition states that fluid "sticks" to solid surfaces. Two immiscible layers of Newtonian fluid are dragged along by the motion of an upper plate as shown in the figure. The bottom plate is stationary and the velocity profiles for each fluid are linear. The top fluid (fluid 1), with a specific gravity of 0.8 and kinematic viscosity of 1.0 cSt, puts a shear stress on the upper plate, and the lower fluid (fluid 2), with a specific gravity of 1.1 and kinematic viscosity of 1.3 cSt, puts a shear stress on the bottom plate. Determine the ratio of the shear stress on the top plate to the shear stress on the bottom plate.



SOLUTION:

The shear stress acting on either plate may be found using the expression relating the shear stress to the velocity gradient for a Newtonian fluid:



$$\tau_{yx\,\text{on fluid}} = \mu \frac{\partial u_x}{\partial y} \tag{1}$$

For the bottom plate:

$$\tau_{yx\,\text{on bottom plate}} = -\tau_{yx\,\text{on fluid}}\Big|_{y=0} = -\mu_2 \frac{\Delta u_2}{\Delta y_2} \tag{2}$$

where the derivative in Eqn. (1) may be replace with $\Delta u/\Delta y$ since the velocity profile is linear, μ_2 is the dynamic viscosity of fluid 2, and Δu_2 and Δy_2 are the change in the velocity over the change in y-position, respectively, in fluid 2. Following a similar approach for the top plate gives:

$$\tau_{yx\,\text{on top plate}} = -\tau_{yx\,\text{on fluid}}\Big|_{y=H} = -\mu_1 \frac{\Delta u_1}{\Delta y_1} \tag{3}$$

Taking the ratio of Eqn. (3) to Eqn. (2) gives:

$$\frac{\tau_{yx\,\text{on top plate}}}{\tau_{yx\,\text{on bottom plate}}} = \frac{\mu_1}{\mu_2}\frac{\Delta u_1}{\Delta u_2}\frac{\Delta y_2}{\Delta y_1} \tag{4}$$

Note that the dynamic viscosity is related to the specific gravity, SG, and kinematic viscosity, ν, by:

$$\mu = \rho\nu = SG\rho_{H_2O}\nu \tag{5}$$

where ρ is the fluid density and ρ_{H2O} is the density of water. Substituting Eqn. (5) into Eqn. (4) gives:

$$\frac{\tau_{yx\,\text{on top plate}}}{\tau_{yx\,\text{on bottom plate}}} = \frac{SG_1}{SG_2}\frac{\nu_1}{\nu_2}\frac{\Delta u_1}{\Delta u_2}\frac{\Delta y_2}{\Delta y_1} \tag{6}$$

Using the given values:

$SG_1 = 0.8$ $\quad SG_2 = 1.1$
$\nu_1 = 1.0$ cSt $\quad \nu_2 = 1.3$ cSt
$\Delta u_1 = (3 - 2)$ m/s $= 1$ m/s
$\Delta u_2 = (2 - 0)$ m/s $= 2$ m/s
$\Delta y_1 = 0.010$ m
$\Delta y_2 = 0.036$ m

$$\Rightarrow \boxed{\frac{\tau_{yx\,\text{on top plate}}}{\tau_{yx\,\text{on bottom plate}}} = 1.0}$$

Note that the stress across a fluid interface is continuous and since the shear stress is constant in the fluid, the shear stresses on the plates should be identical.