The estimated dimensions of a soda can are $D \approx 66.0$ mm and $H \approx 110$ mm. Determine the accuracy with which the diameter and height must be measured to estimate the volume of the can within an uncertainty of ±0.5%.

SOLUTION:

The volume of a cylinder (*e.g.* the soda can) is:

$$V = \tfrac{\pi}{4} D^2 H \tag{1}$$

The relative uncertainty in V is:

$$u_V = \left[u_{V,D}^2 + u_{V,H}^2 \right]^{1/2} \tag{2}$$

where

$$u_{V,D} = \frac{1}{V}\frac{\partial V}{\partial D}\delta D = \frac{4}{\pi D^2 H}\left(\frac{2\pi DH}{4}\right)\delta D = 2\frac{\delta D}{D} = 2u_D \tag{3}$$

$$u_{V,H} = \frac{1}{V}\frac{\partial V}{\partial H}\delta H = \frac{4}{\pi D^2 H}\left(\frac{\pi D^2}{4}\right)\delta H = \frac{\delta H}{H} = u_H \tag{4}$$

Substitute into Eqn. (2).

$$u_V = \left[4u_D^2 + u_H^2 \right]^{1/2} \tag{5}$$

Express the right-hand side of the previous equation in terms of absolute uncertainties and re-arrange to solve for the absolute uncertainty in the diameter and height measurements.

$$u_V^2 = 4\left(\frac{\delta x}{D}\right)^2 + \left(\frac{\delta x}{H}\right)^2 \tag{6}$$

$$u_V^2 = \left(\frac{4}{D^2} + \frac{1}{H^2}\right)(\delta x)^2 \tag{7}$$

$$\boxed{\therefore \delta x = u_V \left(\frac{4}{D^2} + \frac{1}{H^2}\right)^{-1/2}} \tag{8}$$

Since we wish to measure the volume to within a relative uncertainty of $u_V = 0.005$, and $D = 66.0$ mm and $H = 110$ mm, we must have a length measurement precision of $\boxed{\delta x = 0.158 \text{ mm}}$.