An engineer wishes to determine the efficiency with which paint is applied to a sample surface using a particular spray nozzle. The mass deposition efficiency, *MDE*, is defined as:

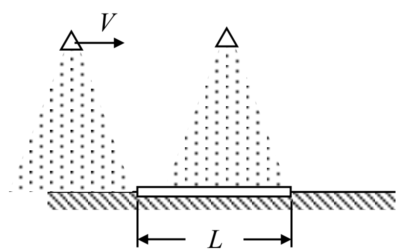
$$MDE \equiv \frac{m_f - m_i}{m_a}$$

where m_f is the mass of the surface after painting and drying, m_i is the initial mass of the surface (no paint applied), and m_a is the mass of paint that came out of the spray nozzle in a specified period of time. The mass of paint from the spray nozzle, m_a, may be calculated using:

$$m_a = \dot{m}Ts$$

where $\dot{m}$ is the mass flow rate through the nozzle, T is the duration that the spray is applied to the surface, and s is the percentage of (paint) solids present in the spray. The paint is applied by traversing the nozzle over the surface, with a traverse distance, L, at a constant speed, V, as shown in the figure below. Hence, the duration T may be found from:

$$T = \frac{L}{V}$$



Given the following uncertainties:

$\delta\dot{m}$ = ±0.025 kg/min
δs = ±2%
δL = ±1.5 mm
δV = ±0.5 mm/sec
δm_f = ±0.0001 kg
δm_i = ±0.0001 kg
δm_f = ±0.0001 kg

determine the mass deposition efficiencies, *MDE*s, with uncertainties for the following cases.

$\dot{m}$ [kg/min]	V [m/s]	L [m]	m_f [kg]	m_i [kg]	s [%]
0.92	0.127	0.304	0.0498	0.0341	51.2
1.79	0.254	0.306	0.0502	0.0339	51.0
1.66	0.254	0.302	0.0523	0.0368	50.9

SOLUTION:

First determine the relative uncertainty in the duration, *T*.

$$u_T = \frac{\delta T}{T} = \sqrt{u_{T,L}^2 + u_{T,V}^2} \tag{1}$$

where

$$u_{T,L} = \frac{1}{T}\frac{\partial T}{\partial L}\delta L = \left(\frac{V}{L}\right)\left(\frac{1}{V}\right)(\delta L) = \frac{\delta L}{L} = u_L \tag{2}$$

$$u_{T,V} = \frac{1}{T}\frac{\partial T}{\partial V}\delta V = \left(\frac{V}{L}\right)\left(-\frac{L}{V^2}\right)(\delta V) = -\frac{\delta V}{V} = -u_V \tag{3}$$

$$\therefore u_T = \sqrt{u_L^2 + u_V^2} \tag{4}$$

Now determine the relative uncertainty in the applied mass, m_a:

$$u_{m_a} = \frac{\delta m_a}{m_a} = \sqrt{u_{m_a,\dot{m}}^2 + u_{m_a,T}^2 + u_{m_a,s}^2} \tag{5}$$

where

$$u_{m_a,\dot{m}} = \frac{1}{m_a}\frac{\partial m_a}{\partial \dot{m}}\delta \dot{m} = \left(\frac{1}{\dot{m}Ts}\right)(Ts)(\delta \dot{m}) = \frac{\delta \dot{m}}{\dot{m}} = u_{\dot{m}} \tag{6}$$

$$u_{m_a,T} = \frac{1}{m_a}\frac{\partial m_a}{\partial T}\delta T = \left(\frac{1}{\dot{m}Ts}\right)(\dot{m}s)(\delta T) = \frac{\delta T}{T} = u_T \tag{7}$$

$$u_{m_a,s} = \frac{1}{m_a}\frac{\partial m_a}{\partial s}\delta s = \left(\frac{1}{\dot{m}Ts}\right)(\dot{m}T)(\delta s) = \frac{\delta s}{s} = u_s \tag{8}$$

$$\therefore u_{m_a} = \sqrt{u_{\dot{m}}^2 + u_T^2 + u_s^2} \tag{9}$$

Lastly, determine the relative uncertainty in the mass deposition efficiency, *MDE*:

$$u_{MDE} = \frac{\delta(MDE)}{MDE} = \sqrt{u_{MDE,m_f}^2 + u_{MDE,m_i}^2 + u_{MDE,m_a}^2} \tag{10}$$

where

$$u_{MDE,m_f} = \frac{1}{MDE}\frac{\partial(MDE)}{\partial m_f}\delta m_f = \left(\frac{m_a}{m_f - m_i}\right)\left(\frac{1}{m_a}\right)(\delta m_f) = \frac{\delta m_f}{m_f - m_i} = \frac{\delta m_f / m_f}{1 - m_i / m_f} = \frac{u_{m_f}}{1 - m_i / m_f} \tag{11}$$

$$u_{MDE,m_i} = \frac{1}{MDE}\frac{\partial(MDE)}{\partial m_i}\delta m_i = \left(\frac{m_a}{m_f - m_i}\right)\left(\frac{-1}{m_a}\right)(\delta m_i) = \frac{-\delta m_i}{m_f - m_i} = \frac{-\delta m_i / m_i}{m_f / m_i - 1} = \frac{u_{m_i}}{1 - m_f / m_i} \tag{12}$$

$$u_{MDE,m_a} = \frac{1}{MDE}\frac{\partial(MDE)}{\partial m_a}\delta m_a = \left(\frac{m_a}{m_f - m_i}\right)\left[\frac{-(m_f - m_i)}{m_a^2}\right](\delta m_a) = \frac{-\delta m_a}{m_a} = -u_{m_a} \tag{13}$$

$$\therefore u_{MDE} = \sqrt{\frac{u_{m_f}^2}{\left(1 - m_i / m_f\right)^2} + \frac{u_{m_i}^2}{\left(1 - m_f / m_i\right)^2} + u_{m_a}^2} \tag{14}$$

Create a spreadsheet to perform the calculations.

mdot [kg/min]	u_{mdot}	V [m/s]	u_V	L [m]	u_L	m_f [kg]	u_{mf}	m_i [kg]	u_{mi}	s	u_s	T [s]	u_T	m_a [kg]	u_{ma}	MDE	u_{MDE}
0.92	2.7%	0.127	0.4%	0.304	0.5%	0.0498	0.2%	0.0341	0.3%	51.2%	3.9%	2.39	0.6%	0.0188	4.8%	83.5%	4.9%
1.79	1.4%	0.254	0.2%	0.306	0.5%	0.0502	0.2%	0.0339	0.3%	51.0%	3.9%	1.20	0.5%	0.0183	4.2%	88.9%	4.3%
1.66	1.5%	0.254	0.2%	0.302	0.5%	0.0523	0.2%	0.0368	0.3%	50.9%	3.9%	1.19	0.5%	0.0167	4.2%	92.6%	4.3%

Another approach to determining the uncertainties is to substitute the supporting formulas directly into the expression for the mass deposition efficiency.

$$MDE = \frac{m_f - m_i}{\dot{m}\left(L/V\right)s} \tag{15}$$

$$u_{MDE} = \frac{\delta(MDE)}{MDE} = \sqrt{u_{MDE,m_f}^2 + u_{MDE,m_i}^2 + u_{MDE,\dot{m}}^2 + u_{MDE,L}^2 + u_{MDE,V}^2 + u_{MDE,s}^2} \tag{16}$$

where

$$u_{MDE,m_f} = \frac{1}{MDE}\frac{\partial(MDE)}{\partial m_f}\delta m_f = \left(\frac{\dot{m}\left(L/V\right)s}{m_f - m_i}\right)\left(\frac{1}{\dot{m}\left(L/V\right)s}\right)(\delta m_f) = \frac{\delta m_f}{m_f - m_i} = \frac{\delta m_f / m_f}{1 - m_i/m_f} = \frac{u_{m_f}}{1 - m_i/m_f} \tag{17}$$

$$u_{MDE,m_i} = \frac{1}{MDE}\frac{\partial(MDE)}{\partial m_i}\delta m_i = \left(\frac{\dot{m}\left(L/V\right)s}{m_f - m_i}\right)\left(\frac{-1}{\dot{m}\left(L/V\right)s}\right)(\delta m_i) = \frac{-\delta m_i}{m_f - m_i} = \frac{-\delta m_i / m_i}{m_f/m_i - 1} = \frac{u_{m_i}}{1 - m_f/m_i} \tag{18}$$

$$u_{MDE,\dot{m}} = \frac{1}{MDE}\frac{\partial(MDE)}{\partial \dot{m}}\delta \dot{m} = \left(\frac{\dot{m}\left(L/V\right)s}{m_f - m_i}\right)\left[\frac{-(m_f - m_i)}{\dot{m}^2\left(L/V\right)s}\right](\delta \dot{m}) = \frac{-\delta \dot{m}}{\dot{m}} = -u_{\dot{m}} \tag{19}$$

$$u_{MDE,L} = \frac{1}{MDE}\frac{\partial(MDE)}{\partial L}\delta L = \left(\frac{\dot{m}\left(L/V\right)s}{m_f - m_i}\right)\left[\frac{-(m_f - m_i)}{\dot{m}\left(L^2/V\right)s}\right](\delta L) = \frac{-\delta L}{L} = -u_L$$

$$u_{MDE,V} = \frac{1}{MDE}\frac{\partial(MDE)}{\partial V}\delta V = \left(\frac{\dot{m}\left(L/V\right)s}{m_f - m_i}\right)\left[\frac{(m_f - m_i)}{\dot{m}Ls}\right](\delta V) = \frac{\delta V}{V} = u_V$$

$$u_{MDE,s} = \frac{1}{MDE}\frac{\partial(MDE)}{\partial s}\delta s = \left(\frac{\dot{m}\left(L/V\right)s}{m_f - m_i}\right)\left[\frac{-(m_f - m_i)}{\dot{m}\left(L/V\right)s^2}\right](\delta s) = \frac{-\delta s}{s} = -u_s$$

$$\therefore u_{MDE} = \sqrt{\frac{u_{m_f}^2}{\left(1 - m_i/m_f\right)^2} + \frac{u_{m_i}^2}{\left(1 - m_f/m_i\right)^2} + u_{\dot{m}}^2 + u_L^2 + u_V^2 + u_s^2} \tag{20}$$

The uncertainties calculated using Eqn. (20) are exactly the same as those found using Eqn. (14) (this can be proven by simply substituting in for the relative uncertainty expressions in Eqn. (14)).