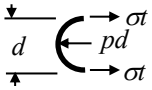
The hoop stress, σ, in a thin-walled cylindrical pressure vessel may be estimated using:

$$\sigma = \frac{pd}{2t}$$



where p is the cylinder's interior gage pressure, d is the cylinder diameter, and t is the vessel wall thickness. The pressure in the vessel is measured to be 30 ± 2 psig, the tank diameter is 2.45 ± 0.03 in., and the wall thickness is 0.0050 ± 0.0002 in.

a. Determine the hoop stress including its uncertainty.
b. Which measurement should be improved first in order to reduce the uncertainty in the hoop stress?

SOLUTION:

The relative uncertainty in σ is:

$$u_\sigma = \left[u_{\sigma,p}^2 + u_{\sigma,d}^2 + u_{\sigma,t}^2 \right]^{1/2} \tag{1}$$

where

$$u_{\sigma,p} = \frac{1}{\sigma}\frac{\partial \sigma}{\partial p}\delta p = \frac{2t}{pd}\left(\frac{d}{2t}\right)\delta p = \frac{\delta p}{p} = u_p \tag{2}$$

$$u_{\sigma,d} = \frac{1}{\sigma}\frac{\partial \sigma}{\partial d}\delta d = \frac{2t}{pd}\left(\frac{p}{2t}\right)\delta d = \frac{\delta d}{d} = u_d \tag{3}$$

$$u_{\sigma,t} = \frac{1}{\sigma}\frac{\partial \sigma}{\partial t}\delta t = \frac{2t}{pd}\left(-\frac{pd}{2t^2}\right)\delta t = -\frac{\delta t}{t} = -u_t \tag{4}$$

Substitute into Eqn. (1).

$$u_\sigma = \left[u_p^2 + u_d^2 + u_t^2 \right]^{1/2} \tag{5}$$

The relative uncertainties in the pressure, diameter, and thickness are:

$$u_p = \frac{\delta p}{p} = \frac{2 \text{ psi}}{30 \text{ psi}} = 6.7\% \tag{6}$$

$$u_d = \frac{\delta d}{d} = \frac{0.03 \text{ in.}}{2.45 \text{ in.}} = 1.2\% \tag{7}$$

$$u_t = \frac{\delta t}{t} = \frac{0.0002 \text{ in.}}{0.005 \text{ in.}} = 4.0\% \tag{8}$$

$$\Rightarrow u_\sigma = 7.9\% \tag{9}$$

$$\boxed{\therefore \sigma = 7350 \pm 580 \text{ psi}}$$

Since the relative uncertainty in the pressure measurement is the greatest, an attempt should be made to improve the accuracy of this measurement first.