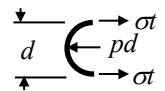


The hoop stress,  $\sigma$ , in a thin-walled cylindrical pressure vessel may be estimated using:

$$\sigma = \frac{pd}{2t}$$


The diagram shows a cross-section of a cylindrical vessel. A central horizontal line represents the diameter, labeled 'd'. Two curved arrows pointing towards each other from the center represent the internal pressure, labeled 'pd'. Two horizontal arrows pointing outwards from the wall represent the hoop stress, labeled 'sigma t'.

where  $p$  is the cylinder's interior gage pressure,  $d$  is the cylinder diameter, and  $t$  is the vessel wall thickness. The pressure in the vessel is measured to be  $30 \pm 2$  psig, the tank diameter is  $2.45 \pm 0.03$  in., and the wall thickness is  $0.0050 \pm 0.0002$  in.

- Determine the hoop stress including its uncertainty.
- Which measurement should be improved first in order to reduce the uncertainty in the hoop stress?

SOLUTION:

The relative uncertainty in  $\sigma$  is:

$$u_{\sigma} = \left[ u_{\sigma,p}^2 + u_{\sigma,d}^2 + u_{\sigma,t}^2 \right]^{1/2} \quad (1)$$

where

$$u_{\sigma,p} = \frac{1}{\sigma} \frac{\partial \sigma}{\partial p} \delta p = \frac{2t}{pd} \left( \frac{d}{2t} \right) \delta p = \frac{\delta p}{p} = u_p \quad (2)$$

$$u_{\sigma,d} = \frac{1}{\sigma} \frac{\partial \sigma}{\partial d} \delta d = \frac{2t}{pd} \left( \frac{p}{2t} \right) \delta d = \frac{\delta d}{d} = u_d \quad (3)$$

$$u_{\sigma,t} = \frac{1}{\sigma} \frac{\partial \sigma}{\partial t} \delta t = \frac{2t}{pd} \left( -\frac{pd}{2t^2} \right) \delta t = -\frac{\delta t}{t} = -u_t \quad (4)$$

Substitute into Eqn. (1).

$$u_{\sigma} = \left[ u_p^2 + u_d^2 + u_t^2 \right]^{1/2} \quad (5)$$

The relative uncertainties in the pressure, diameter, and thickness are:

$$u_p = \frac{\delta p}{p} = \frac{2 \text{ psi}}{30 \text{ psi}} = 6.7\% \quad (6)$$

$$u_d = \frac{\delta d}{d} = \frac{0.03 \text{ in.}}{2.45 \text{ in.}} = 1.2\% \quad (7)$$

$$u_t = \frac{\delta t}{t} = \frac{0.0002 \text{ in.}}{0.005 \text{ in.}} = 4.0\% \quad (8)$$

$$\Rightarrow u_{\sigma} = 7.9\% \quad (9)$$

$$\boxed{\therefore \sigma = 7350 \pm 580 \text{ psi}}$$

Since the relative uncertainty in the pressure measurement is the greatest, an attempt should be made to improve the accuracy of this measurement first.