In pneumatic conveying, solid particles such as flour or coal are carried through a duct by a moving air stream. Solids density at any duct location can be measured by passing a laser beam of known intensity, I_0, through the duct and measuring the light intensity transmitted to the other side, I. A transmission factor is found using:

$$T = \frac{I}{I_0} = \exp\left(-KEW\right) \quad \text{where } 0 \le T \le 1$$

Here W is the width of the duct, K is the solids density, and E is a factor taken as 2.0±0.4 kg/m^2 for spheroidal particles. Determine how the relative uncertainty in K is related to the relative uncertainties of the other variables. If the transmission factor and duct width can be measured to within ±1%, can the solids density be measured to within 5%? 10%? Discuss your answer remembering that T varies from 0 to 1.

SOLUTION:

Solve for the solids density using the definition of the transmission factor.

$$T = \exp(-KEW)$$

$$K = \frac{-1}{EW}\ln T \tag{1}$$

The relative uncertainty in the solids density is given by:

$$u_K = \left[u_{K,E}^2 + u_{K,W}^2 + u_{K,T}^2\right]^{1/2} \tag{2}$$

where

$$u_{K,E} = \frac{1}{K}\frac{\partial K}{\partial E}\delta E = \left(\frac{-EW}{\ln T}\right)\left(\frac{\ln T}{E^2 W}\right)\delta E = -\frac{\delta E}{E} = -u_E \tag{3}$$

$$u_{K,W} = \frac{1}{K}\frac{\partial K}{\partial W}\delta W = \left(-\frac{EW}{\ln T}\right)\left(\frac{\ln T}{EW^2}\right)\delta W = -\frac{\delta W}{W} = -u_W \tag{4}$$

$$u_{K,T} = \frac{1}{K}\frac{\partial K}{\partial T}\delta T = \left(-\frac{EW}{\ln T}\right)\left(\frac{-1}{EWT}\right)\delta T = \frac{\delta T}{T\ln T} = \frac{u_T}{\ln T} \tag{5}$$

Substitute into Eqn. (2).

$$u_K = \left[u_E^2 + u_W^2 + \left(\frac{u_T}{\ln T}\right)^2\right]^{1/2} \tag{6}$$

where the relative uncertainties are:

$$u_E = \frac{\delta E}{E} = \frac{0.4}{2.0} = 20\% \tag{7}$$

$$u_W = 1\% \tag{8}$$

$$u_T = 1\% \tag{9}$$

Recall that $0 \le T \le 1$ so that:

$$T = 0: \quad \lim_{T\to 0}(\ln T) = -\infty \quad \Rightarrow \quad \lim_{T\to 0}\left(\frac{u_T}{\ln T}\right) = 0 \quad \Rightarrow \quad \lim_{T\to 0}(u_K) = 20\% \tag{10}$$

$$T = 1: \quad \ln T\big|_{T=1} = 0 \quad \Rightarrow \quad \lim_{T\to 1}\left(\frac{u_T}{\ln T}\right) = \infty \quad \Rightarrow \quad \lim_{T\to 1}(u_K) = \infty \tag{11}$$

$$\Rightarrow \quad 20\% \le u_K \le \infty \tag{12}$$

Hence, it is not possible to measure K to within either 5% or 10%. In fact, it is not possible to measure K to better than 20% relative uncertainty.