A certain obstruction-type flowmeter is used to measure the flow of air at low velocities. The relation describing the flow rate is:

$$\dot{m} = CA\left[\frac{2p_1}{RT_1}(p_1 - p_2)\right]^{1/2}$$

where C is an empirical discharge coefficient, A is the flow area, p_1 and p_2 are the upstream and downstream pressures, T_1 is the upstream temperature, and R is the gas constant for air.

Calculate the relative uncertainty in the mass flow rate for the following conditions:

$C = 0.92 \pm 0.005$ (from calibration data)
$p_1 = 25 \pm 0.5$ psia
$T_1 = 530 \pm 2$ °R
$\Delta p = p_1 - p_2 = 1.4 \pm 0.005$ psia
$A = 1.0 \pm 0.001$ in^2

What factors contribute the most to the uncertainty in the mass flow rate?

SOLUTION:

The relative uncertainty in the mass flow rate is given by:

$$u_{\dot{m}} = \left[u_{\dot{m},C}^2 + u_{\dot{m},A}^2 + u_{\dot{m},p_1}^2 + u_{\dot{m},T_1}^2 + u_{\dot{m},\Delta p}^2 \right]^{1/2} \tag{1}$$

where

$$u_{\dot{m},C} = \frac{1}{\dot{m}} \frac{\partial \dot{m}}{\partial C} \delta C = \frac{\delta C}{C} = u_C \tag{2}$$

$$u_{\dot{m},A} = \frac{1}{\dot{m}} \frac{\partial \dot{m}}{\partial A} \delta A = \frac{\delta A}{A} = u_A \tag{3}$$

$$u_{\dot{m},p_1} = \frac{1}{\dot{m}} \frac{\partial \dot{m}}{\partial p_1} \delta p_1 = \frac{1}{2} \frac{\delta p_1}{p_1} = \tfrac{1}{2} u_{p_1} \tag{4}$$

$$u_{\dot{m},T_1} = \frac{1}{\dot{m}} \frac{\partial \dot{m}}{\partial T_1} \delta T_1 = -\frac{1}{2} \frac{\delta T_1}{T_1} = -\tfrac{1}{2} u_{T_1} \tag{5}$$

$$u_{\dot{m},\Delta p} = \frac{1}{\dot{m}} \frac{\partial \dot{m}}{\partial \Delta p} \delta\left(\Delta p\right) = \frac{1}{2} \frac{\delta\left(\Delta p\right)}{\Delta p} = \tfrac{1}{2} u_{\Delta p} \tag{6}$$

Note that the there is negligible uncertainty in the gas constant *R* since it is presumed to be known to a high degree of accuracy.

Substitute into Eqn. (1).

$$u_{\dot{m}} = \left[u_C^2 + u_A^2 + \tfrac{1}{4}u_{p_1}^2 + \tfrac{1}{4}u_{T_1}^2 + \tfrac{1}{4}u_{\Delta p}^2\right]^{1/2} \quad (7)$$

where the relative uncertainties are:

$$u_C = \frac{\delta C}{C} = \frac{0.005}{0.92} = 0.54\% \quad (8)$$

$$u_A = \frac{\delta A}{A} = \frac{0.001\ \text{in}^2}{1.0\ \text{in}^2} = 0.10\% \quad (9)$$

$$u_{p_1} = \frac{\delta p_1}{p_1} = \frac{0.5\ \text{psia}}{25\ \text{psia}} = 2.0\% \quad (10)$$

$$u_{T_1} = \frac{\delta T_1}{T_1} = \frac{2\ \ \text{R}}{530\ \ \text{R}} = 0.38\% \quad (11)$$

$$u_{\Delta p} = \frac{\delta(\Delta p)}{\Delta p} = \frac{0.005\ \text{psia}}{1.4\ \text{psia}} = 0.36\% \quad (12)$$

$$\Rightarrow \boxed{u_{\dot{m}} = 1.2\%}$$

Examine the contributions of each term on the right hand side of Eqn. (7) to determine which uncertainty has the greatest influence on the uncertainty in $\dot{m}$.

$$u_C^2 = \left(5.4*10^{-3}\right)^2 = 2.9*10^{-5}$$

$$u_A^2 = \left(1.0*10^{-3}\right)^2 = 1.0*10^{-6}$$

$$\tfrac{1}{4}u_{p_1}^2 = \tfrac{1}{4}\left(2.0*10^{-2}\right)^2 = 1.0*10^{-4}$$

$$\tfrac{1}{4}u_{T_1}^2 = \tfrac{1}{4}\left(3.8*10^{-3}\right)^2 = 3.6*10^{-6}$$

$$\tfrac{1}{4}u_{\Delta p}^2 = \tfrac{1}{4}\left(3.6*10^{-3}\right)^2 = 3.2*10^{-6}$$

The uncertainty in the p_1 measurement contributes the most to the uncertainty in $\dot{m}$.