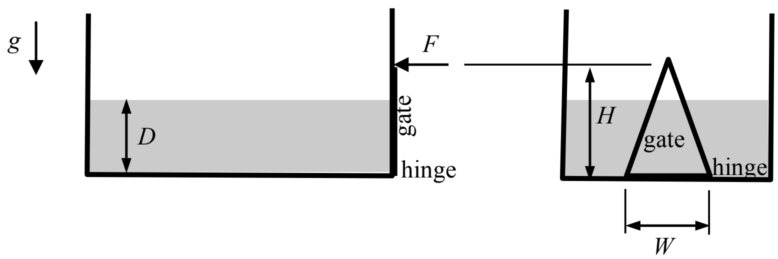
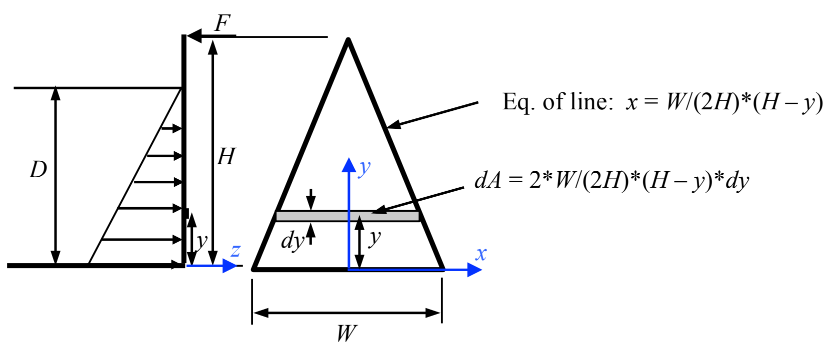
The tank shown below is partially filled with a liquid of density *r* and is open to the atmosphere. A triangular gate is ninged at the bottom and held closed by a force applied at the top. Determine the force *F* in terms of the liquid density *ρ*, the acceleration due to gravity *g*, the liquid depth *D*, the gate height *H*, and the gate width *W*.



SOLUTION:
Balance moments on the gate. Since the pressure varies over the surface of the gate and because the gate width changes with depth, we'll need to integrate the pressure force over the gate surface. To do this, divide the gate into small areas over which the pressure remains constant.



Balance moments about the hinge,

$$\sum \mathbf{M}_{\text{hinge}} = \mathbf{0} = \left(H\hat{\mathbf{j}} \times -F\hat{\mathbf{k}}\right) + \int_A \underbrace{y\hat{\mathbf{j}}}_{\substack{\text{moment}\\\text{arm}}} \times \underbrace{d\mathbf{F}_p}_{\substack{\text{pressure}\\\text{force}}}, \tag{1}$$

where,

$$d\mathbf{F}_p = -p\,d\mathbf{A}, \tag{2}$$

$$d\mathbf{A} = 2\left(\frac{W}{2H}\right)\left(H-y\right)dy\left(-\hat{\mathbf{k}}\right), \tag{3}$$

$$p_{\text{gage}} = \rho g\left(D-y\right). \tag{4}$$

Substitute and solve for *F*,

$$\mathbf{0} = HF\hat{\mathbf{i}} + \int_{y=0}^{y=D} y\rho g\left(D-y\right)2\left(\frac{W}{2H}\right)\left(H-y\right)dy\left(-\hat{\mathbf{i}}\right), \tag{5}$$

$$HF = \rho g\left(\frac{W}{H}\right)\int_{y=0}^{y=D} y\left(D-y\right)\left(H-y\right)dy = \rho g\left(\frac{W}{H}\right)\int_{y=0}^{y=D}\left[DHy - \left(D+H\right)y^2 + y^3\right]dy, \tag{6}$$

$$F = \rho g\left(\frac{W}{H^2}\right)\left[\tfrac{1}{2}D^3H - \tfrac{1}{3}\left(D+H\right)D^3 + \tfrac{1}{4}D^4\right], \tag{7}$$

$$F = \rho g\left(\frac{W}{H^2}\right)\left(\tfrac{1}{2}D^3H - \tfrac{1}{3}D^4 - \tfrac{1}{3}HD^3 + \tfrac{1}{4}D^4\right), \tag{8}$$

$$\boxed{F = \frac{1}{6}\rho g\left(\frac{WD^3}{H^2}\right)\left(H - \frac{1}{2}D\right)}. \tag{9}$$