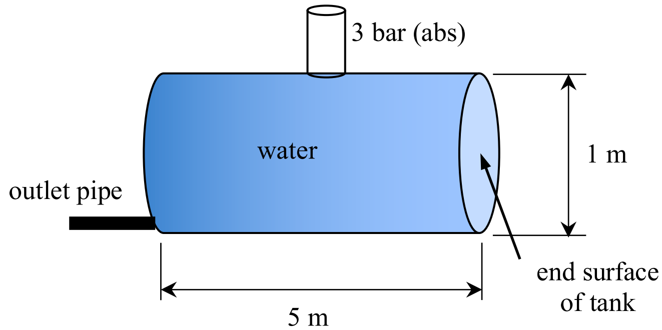
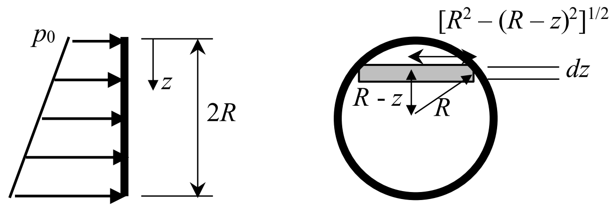
A cylindrical tank if filled with water. In order to control the flow rate from the tank, a pressure can be applied to the water surface by a compressor. For an applied absolute pressure of 3 bar, calculate the hydrostatic force exerted by the water on the end surface of the tank.



SOLUTION:
Draw the pressure distribution acting on the tank end surface due to the water in the tank.



The hydrostatic pressure force on the tank surface due to the water is,

$$F = \int_{z=0}^{z=2R} p\,dA = \int_{z=0}^{z=2R} \underbrace{\left(p_0 + \rho g z\right)}_{=p} \underbrace{\left[2\sqrt{R^2 - \left(R-z\right)^2}\,dz\right]}_{=dA}, \tag{1}$$

$$F = 2\int_{z=0}^{z=2R} \left(p_0 + \rho g z\right)\sqrt{2Rz - z^2}\,dz = 2\left[p_0 \int_{z=0}^{z=2R} \sqrt{2Rz - z^2}\,dz + \rho g \int_{z=0}^{z=2R} z\sqrt{2Rz - z^2}\,dz\right], \tag{2}$$

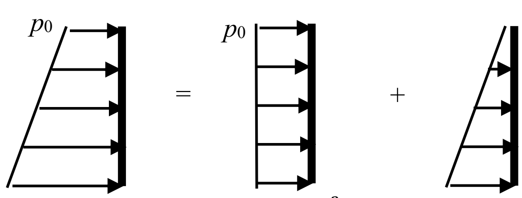
$$F = 2\left[p_0 \frac{\pi R^2}{2} + \rho g \frac{\pi R^3}{2}\right], \tag{3}$$

$$\boxed{F = \pi R^2\left(p_0 + \rho g R\right)}. \tag{4}$$

Using the following parameters:

$R = 0.5$ m
$p_0 = 3$ bar (abs) = 300 kPa (abs)
$\rho = 1000$ kg/m^3
$g = 9.81$ m/s^2
=> $\boxed{F = 23.6 \text{ MN}}$.

Note an alternate approach to solving the problem is to break the applied pressure into a constant part at pressure p_0 and the linearly increasing part, as shown in the figures below.



$$F_1 = p_0 \pi R^2 \qquad F_2 = 2\rho g \int_{z=0}^{z=2R} z\sqrt{2Rz - z^2}\,dz$$