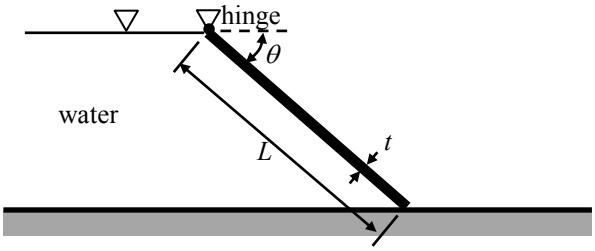
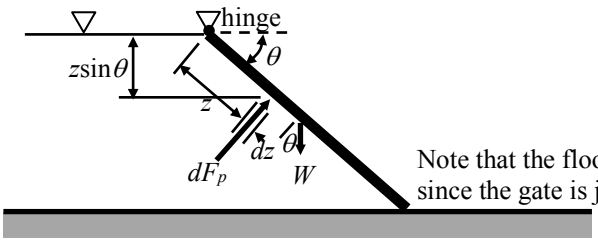
A plane gate of uniform thickness t and width into the page w holds back a depth of water as shown. Find the minimum weight of the gate needed to keep the gate closed.



SOLUTION:
Draw a free body diagram of the gate.



Note that the floor exerts no force on the gate since the gate is just about to open.

Sum moments about the gate's hinge, noting that the gate is in equilibrium and just about to open,

$$\sum M_{\text{hinge}} = 0 = -\left(\frac{L}{2}\right)\left(W\cos\theta\right) + \int_{z=0}^{z=L} z\underbrace{\underbrace{\left(\rho g z \sin\theta\right)}_{=p}\underbrace{\left(w dz\right)}_{=dA}}_{=dF_p}, \tag{1}$$

$$\left(\frac{L}{2}\right)\left(W\cos\theta\right) = \rho g w \sin\theta \int_{z=0}^{z=L} z^2\, dz, \tag{2}$$

$$\left(\frac{L}{2}\right)\left(W\cos\theta\right) = \frac{1}{3}\rho g w L^3 \sin\theta, \tag{3}$$

$$\boxed{W = \tfrac{2}{3}\rho g w L^2 \tan\theta}. \tag{4}$$