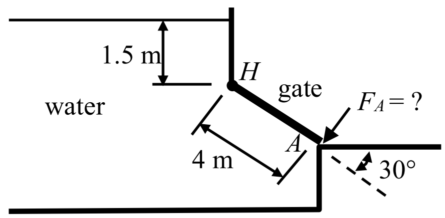
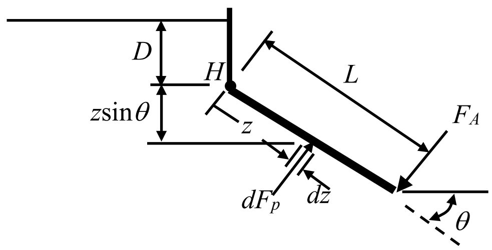
The 3 m wide (into the page) gate shown in the figure is hinged at point *H*. Calculate the force required at point *A* to hold the gate closed



SOLUTION:
Draw a free body diagram of the gate, just as the gate is about to open.



Sum moments about the hinge H and set them equal to zero since the gate isn't accelerating,

$$\sum M_H = 0 = \int_{z=0}^{z=L} z \underbrace{\underbrace{\rho g\left(D + z\sin\theta\right)}_{=p}\underbrace{\left(Tdz\right)}_{=dA}}_{=dF_p} - LF_A\,, \tag{1}$$

where T is the thickness of the gate into the page. The first z in the integral is the moment arm out to the differential hydrostatic pressure force dF_P acting on area $dA = Tdz$. Note that the pressure is a function of the depth from the free surface, $D + z\sin\theta$.

Simplify Eq. (1) and solve for F_A,

$$LF_A = \rho g T \int_{z=0}^{z=L} \left(Dzdz + \sin\theta z^2 dz\right), \tag{2}$$

$$LF_A = \rho g T\left(\tfrac{1}{2}DL^2 + \tfrac{1}{3}L^3\sin\theta\right), \tag{3}$$

$$\boxed{F_A = \rho g T L\left(\tfrac{1}{2}D + \tfrac{1}{3}L\sin\theta\right)}. \tag{4}$$

Using the given data,

$\rho = 1000\ \text{kg/m}^3$,
$g = 9.81\ \text{m/s}^2$,
$T = 3$ m,
$L = 4$ m,
$D = 1.5$ m,
$\theta = 30°$,
=> $\boxed{F_A = 167\ \text{kN}}$