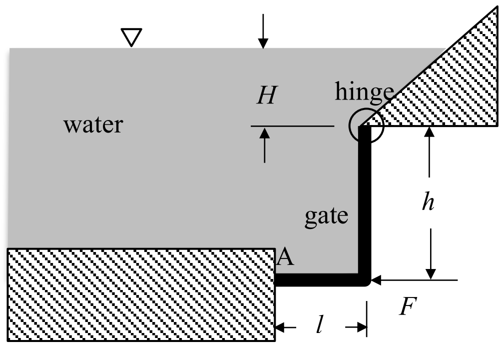
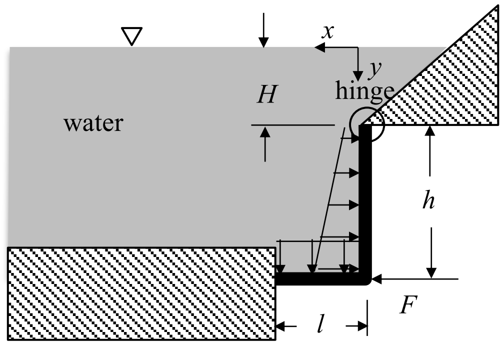
The rigid, L-shaped gate shown in the figure can rotate about the hinge and rests against the rigid support at point A. What is the minimum horizontal force, $F$ required to hold the gate closed if its width is $w = 3$ m and the lengths are $h = 4$ m and $l = 2$ m? The height of the free surface above the hinge is $H = 3$ m. You may neglect the weight of the gate and the friction in the hinge. Note that the back of the gate is exposed to the atmosphere.



SOLUTION:




Balance moments about the hinge,

$$\sum M_{\text{hinge}} = 0 = \int_{y=H}^{y=H+h} \underbrace{(y-H)}_{\text{moment arm length}} \underbrace{\rho g y}_{\text{pressure}} \underbrace{(w\,dy)}_{\text{area}} + \int_{x=0}^{x=l} \underbrace{x}_{\text{moment arm length}} \underbrace{\rho g(H+h)}_{\text{pressure}} \underbrace{(w\,dx)}_{\text{area}} - \underbrace{hF}_{\text{moment due to applied force}}, \tag{1}$$

$$\rho g w \int_{y=H}^{y=H+h} (y-H)\,y\,dy + \rho g(H+h)w \int_{x=0}^{x=l} x\,dx - hF = 0\,, \tag{2}$$

$$hF = \rho g w\left(\tfrac{1}{3}y^3 - \tfrac{1}{2}Hy^2\right)_{y=H}^{y=H+h} + \tfrac{1}{2}\rho g(H+h)wl^2\,, \tag{3}$$

$$hF = \rho g w\left\{\left(\tfrac{1}{3}\left[(H+h)^3 - H^3\right] - \tfrac{1}{2}H\left[(H+h)^2 - H^2\right]\right)\right\} + \tfrac{1}{2}\rho g(H+h)wl^2\,, \tag{4}$$

$$F = \rho g w\left[\tfrac{1}{2}Hh + \tfrac{1}{3}h^2 + \tfrac{1}{2}\left(H/h + 1\right)l^2\right]. \tag{5}$$

Using the given data,

| | |
|---|---|
| $\rho$ | = 1000 kg/m$^3$ |
| □ | = 9.81 m/s$^2$ |
| $w$ | = 3 m |
| $H$ | = 3 m |
| $h$ | = 4 m |
| $l$ | = 2 m |

$\Rightarrow$ $\boxed{F = 437\text{ kN}}$