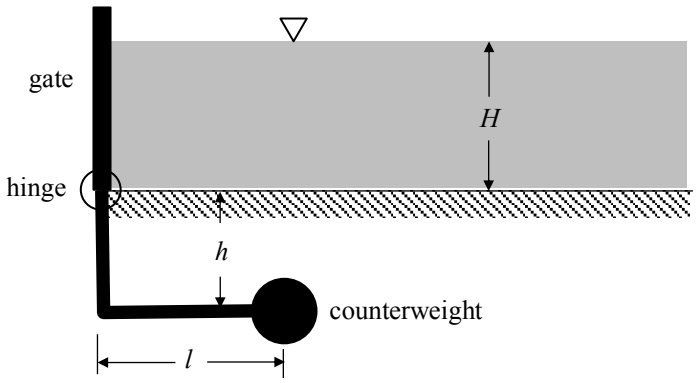
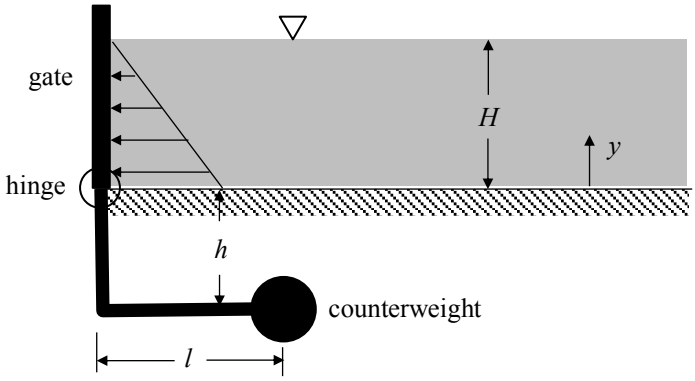
The $w = 4$ ft wide gate shown in the figure pivots about a hinge. The gate is held in place by a counterweight with a weight of $W = 2000$ lb_f, which is located a distance $h = 5$ ft below the base of the water and a distance $l = 3$ ft from the gate. Determine the depth of the water, H, for which the gate remains in the equilibrium position shown. You may assume the gate mass is small compared to the counterweight mass, and that the hinge friction is negligible.



SOLUTION:



Balance moments about the hinge,

$$\sum M_{\text{hinge}} = 0 = \int_{y=0}^{y=H} \underbrace{y}_{\text{moment arm length}} \underbrace{\rho g(H-y)}_{\text{pressure}} \underbrace{(w\,dy)}_{\text{area}} - \underbrace{lW}_{\substack{\text{moment due to} \\ \text{counterweight}}}, \tag{1}$$

$$\rho g w \int_{y=0}^{y=H} y(H-y)\,dy = lW, \tag{2}$$

$$\rho g w \left(\tfrac{1}{2} H y^2 - \tfrac{1}{3} y^3 \right)_{y=0}^{y=H} = lW, \tag{3}$$

$$\tfrac{1}{6} H^3 = \frac{lW}{\rho g w}, \tag{4}$$

$$\boxed{H = \left(\frac{6lW}{\rho g w} \right)^{1/3}}. \tag{5}$$

Using the given data,

ρg = 62.4 $\text{lb}_\text{f}/\text{ft}^3$
W = 2000 lb_f
l = 3 ft
w = 4 ft

$\Rightarrow$ $\boxed{H = 5.2 \text{ ft}}$