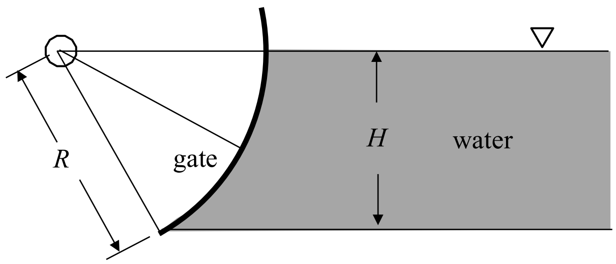
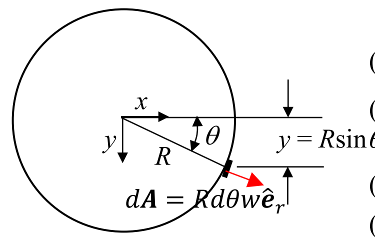
The figure shows a Tainter gate used to control water flow from a dam. The gate radius is R = 20 m, the gate width is w = 35 m, and the water depth is H = 10 m. Determine the force components, magnitude, and line of action of the force that the water exerts on the gate.



SOLUTION:

First determine the force components acting on the gate,



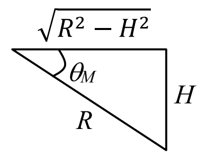
$$\boldsymbol{F} = \int_{y=0}^{y=H} p(-d\boldsymbol{A}) = \int_{y=0}^{y=H} (\rho g y)[-(R d\theta w \hat{\boldsymbol{e}}_r)], \quad (1)$$

$$\boldsymbol{F} = \int_{\theta=0}^{\theta=\theta_M} (\rho g R \sin\theta)(-R d\theta w \hat{\boldsymbol{e}}_r), \quad (2)$$

where,

$$\sin\theta_M = \frac{H}{R} \Rightarrow \theta_M = \sin^{-1}\left(\frac{H}{R}\right), \quad (3)$$

$$\hat{\boldsymbol{e}}_r = \cos\theta\,\hat{\boldsymbol{\imath}} + \sin\theta\,\hat{\boldsymbol{\jmath}}. \quad (4)$$



Substitute and simplify,

$$\boldsymbol{F} = \int_0^{\theta_M} (\rho g R \sin\theta)[-R d\theta w(\cos\theta\,\hat{\boldsymbol{\imath}} + \sin\theta\,\hat{\boldsymbol{\jmath}})], \quad (5)$$

$$\boldsymbol{F} = -\rho g R^2 w \int_0^{\theta_M} (\sin\theta\cos\theta\, d\theta\hat{\boldsymbol{\imath}} + \sin^2\theta\, d\theta\hat{\boldsymbol{\jmath}}), \quad (6)$$

$$\boldsymbol{F} = -\rho g R^2 w \left\{\left(\frac{1}{2}\sin^2\theta_M\right)\hat{\boldsymbol{\imath}} + \left[\frac{1}{2}\theta_M - \frac{1}{4}\sin(2\theta_M)\right]\hat{\boldsymbol{\jmath}}\right\}, \quad (7)$$

$$F_x = -\frac{1}{2}\rho g R^2 w \sin^2\theta_M, \quad (8)$$

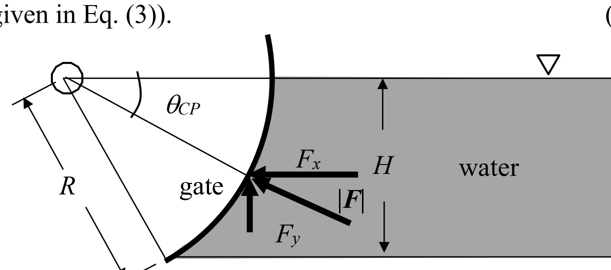
$$F_y = -\frac{1}{2}\rho g R^2 w \left[\theta_M - \frac{1}{2}\sin(2\theta_M)\right], \quad (9)$$

$$F_x = -\frac{1}{2}\rho g R^2 w \left(\frac{H}{R}\right)^2 \Rightarrow F_x = -\frac{1}{2}\rho g H^2 w. \quad (10)$$

$$F_y = -\frac{1}{2}\rho g R^2 w \left[\theta_M - \frac{1}{2}\sin(2\theta_M)\right] \text{ (where } \theta_M \text{ is given in Eq. (3)).} \quad (11)$$

Using the given data,

ρ = 1000 kg/m^3,
g = 9.81 m/s^2,
w = 35 m,
H = 10 m,
R = 20 m,
=> F_x = -17.2 MN and F_y = -6.22 MN



and the force magnitude is |**F**| = 18.3 MN. The angle from the horizontal is,

$$\tan\theta_{CP} = \frac{F_y}{F_x}, \text{ (refer to the figure to the right)} \quad (12)$$

θ_{CP} = 19.9°

Note that the resultant force will pass through the center of the circle (the hinge) since the pressure force acts normal to the surface.