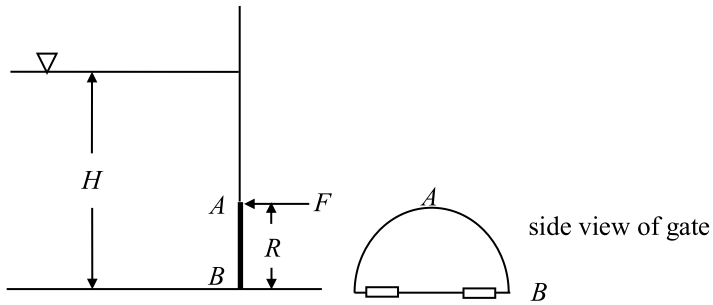
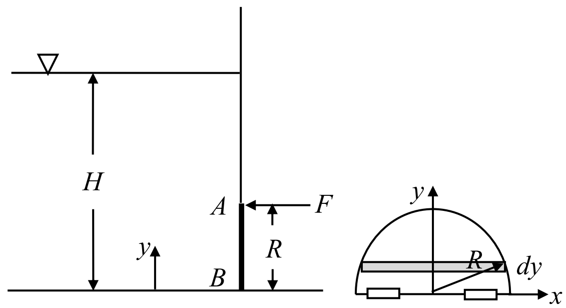
A semi-circular plane gate is hinged along B and held by horizontal force *F* applied at point *A*. The liquid in the reservoir is water. Calculate the minimum force required to hold the gate closed. Hint: An integral table or symbolic algebra software will be helpful in solving the integrals that appear in the derivation.



SOLUTION:



Sum moments about point B.

$$\sum M_B = 0 = RF - \int_{y=0}^{y=R} y p dA \tag{1}$$

$$RF = \int_{y=0}^{y=R} y \underbrace{\rho g (H-y)}_{=p_{\text{gage}}} \underbrace{2\sqrt{R^2-y^2}dy}_{=dA} \tag{2}$$

$$F = \frac{2\rho g}{R} \int_{y=0}^{y=R} y(H-y)\sqrt{R^2-y^2}dy \tag{3}$$

$$F = \frac{2\rho g}{R}\left[H\int_{y=0}^{y=R} y\sqrt{R^2-y^2}dy - \int_{y=0}^{y=R} y^2\sqrt{R^2-y^2}dy\right] \tag{4}$$

Evaluate the integrals using an integral table or symbolic algebra software (*e.g.*, Mathematica).

$$F = \frac{2\rho g}{R}\left[-\frac{1}{3}H\left(R^2-y^2\right)^{3/2}\Big|_{y=0}^{y=R} - \frac{1}{8}\left(y\sqrt{R^2-y^2}\left(2y^2-R^2\right)+R^4\tan^{-1}\left(\frac{y}{\sqrt{R^2-y^2}}\right)\right)\Bigg|_{y=0}^{y=R}\right] \tag{5}$$

$$F = \frac{2\rho g}{R}\left(\frac{1}{3}HR^3 - \frac{1}{8}R^4\frac{\pi}{2}\right) \tag{6}$$

$$\boxed{\therefore F = 2\rho g R^2\left(\frac{1}{3}H - \frac{\pi}{16}R\right)} \tag{7}$$