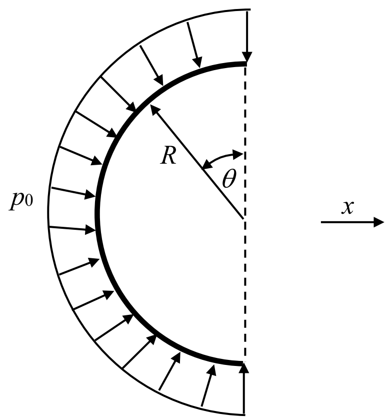
Calculate the net horizontal pressure force acting on the half cylinder shown below. The half cylinder has radius R unit depth into the page, and the gage pressure acting on it is p_0.



SOLUTION:

We can determine the net horizontal pressure force in two ways. The first method directly integrates the horizontal pressure force components over the entire surface and the second method uses the surface's projected area.

Method 1: Integrate the horizontal pressure force components over the entire surface area.

$$dF_{p,x} = \underbrace{p_0 \underbrace{R d\theta}_{=dA}}_{=dF_p} \sin\theta \tag{1}$$

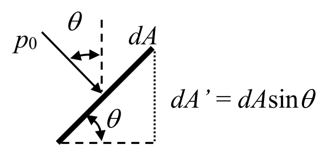
$$F_{p,x} = \int_{\theta=0}^{\theta=\pi} dF_{p,x} = \int_{\theta=0}^{\theta=\pi} p_0 R d\theta \sin\theta = p_0 R \int_{\theta=0}^{\theta=\pi} \sin\theta d\theta = -p_0 R \cos\theta\Big|_0^{\pi} = -p_0 R(-1-1) \tag{2}$$

$$\boxed{\therefore F_{p,x} = p_0(2R)} \tag{3}$$

Method 2: Multiply the pressure with the surface's area projected in the x-direction.

The small amount of horizontal pressure force $dF_{p,x}$ due to the pressure p_0 acting on a small area dA inclined at an angle θ as shown in the figure to the right is,

$$dF_{p,x} = \underbrace{p_0 dA}_{=dF_p} \sin\theta \tag{4}$$



By grouping terms, we see that horizontal pressure force is equivalent to multiplying the pressure by the area projected in the horizontal direction, dA', *i.e.*, the area of the surface viewed from the x-direction.

$$dF_{p,x} = p_0 \underbrace{dA \sin\theta}_{=dA'} \tag{5}$$

Thus, the horizontal pressure force acting on the half-cylinder is simply the pressure multiplied by the cylinder's horizontal projected area, $2R$,

$$\boxed{\therefore F_{p,x} = p_0(2R)} \quad \text{(This is the same result as before!)} \tag{6}$$