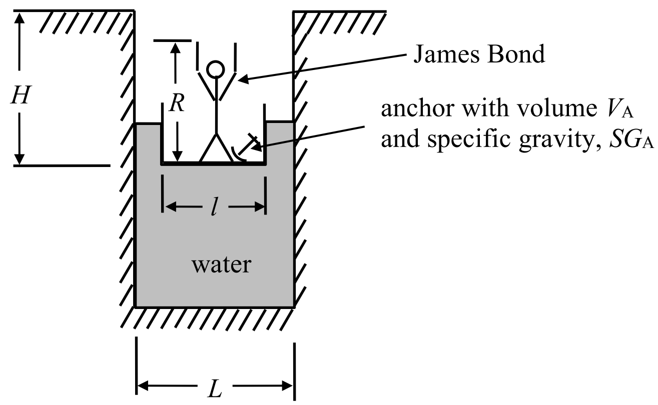
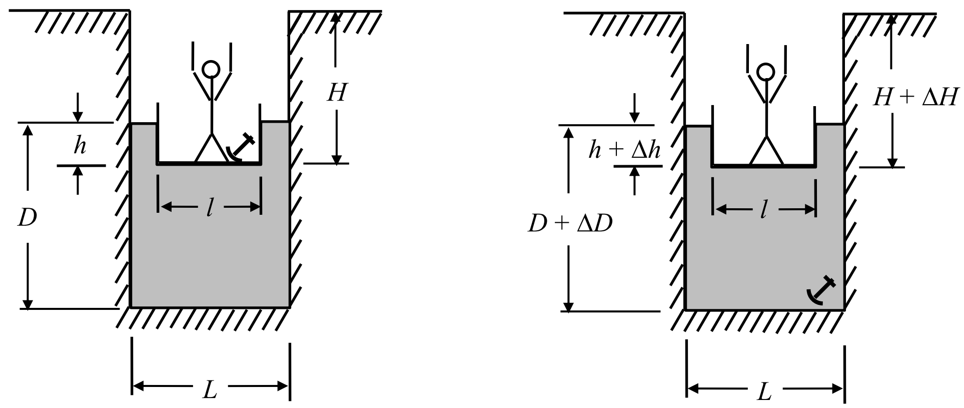
James Bond is trapped on a small raft in a steep walled pit filled with water as shown in the figure. Both the raft and pit have square cross-sections with a side length of $l$=3 ft for the raft and $L$=4 ft for the pit. In the raft there is a steel anchor ($SG_A$=7.85) with a volume of $V_A$=1 ft$^3$. In the current configuration, the distance from the floor of the raft to the top of the pit is $H$=7.5 ft. Unfortunately, Bond can only reach a distance of $R$=7 ft from the floor of the raft. In order for Bond to escape, would it be helpful for him to toss the anchor overboard? Justify your answer with calculations. (Hint: The mass of water is conserved in this problem.)



SOLUTION:

Consider the cases when the anchor is in the raft and out of the raft as shown in the figures below.




(a) (b)

First consider the change in the position of the raft floor relative to the free surface of the water.

Case (a): $$\underbrace{\left(m_{\text{raft+Bond}} + m_{\text{anchor}}\right)g}_{\text{weight of raft \& contents}} = \underbrace{\rho_{H_2O} g l^2 h}_{\text{weight of displaced water}} \tag{1}$$

Case (b): $$\underbrace{\left(m_{\text{raft+Bond}}\right)g}_{\text{weight of raft \& contents}} = \underbrace{\rho_{H_2O} g l^2 \left(h + \Delta h\right)}_{\text{weight of displaced water}} \tag{2}$$

Subtract Eqn. (2) from Eqn. (1) and simplify.

$$\left(m_{\text{raft+Bond}} + m_{\text{anchor}}\right)g - \left(m_{\text{raft+Bond}}\right)g = \rho_{H_2O} g l^2 h - \rho_{H_2O} g l^2 \left(h + \Delta h\right) \tag{3}$$

$$m_{\text{anchor}} = -\rho_{H_2O} l^2 \Delta h \tag{4}$$

$$\Delta h = -\frac{m_{\text{anchor}}}{\rho_{H_2O} l^2} \tag{5}$$

$$\therefore \Delta h = -\frac{SG_{\text{anchor}} V_{\text{anchor}}}{l^2} \tag{6}$$

Note that since $V_{\text{anchor}} > 0$, $\Delta h < 0$ and thus the raft moves up relative to the free surface. However, the free surface will also move so we still don't yet know whether Bond moves up or down relative to the surface of the pit.

We must now consider the movement of the free surface of the water.

Case (a): $$V_{H_2O} = \underbrace{L^2 D}_{\text{volume of H}_2\text{O in pit}} - \underbrace{l^2 h}_{\text{volume of raft in H}_2\text{O}} \tag{7}$$

Case (b): $$V_{H_2O} = \underbrace{L^2 \left(D + \Delta D\right)}_{\text{volume of H}_2\text{O in pit}} - \underbrace{l^2 \left(h + \Delta h\right)}_{\text{volume of raft in H}_2\text{O}} - V_{\text{anchor}} \tag{8}$$

Since the volume of water is conserved, Eqns. (7) and (8) must be equal.

$$L^2 \left(D + \Delta D\right) - l^2 \left(h + \Delta h\right) - V_{\text{anchor}} = L^2 D - l^2 h \tag{9}$$

$$L^2 \Delta D - l^2 \Delta h - V_{\text{anchor}} = 0$$

$$\therefore \Delta D = \frac{l^2 \Delta h + V_{\text{anchor}}}{L^2} \tag{10}$$

$$\therefore \Delta D = \frac{\left(1 - SG_{\text{anchor}}\right) V_{\text{anchor}}}{L^2} \quad \text{(where Eqn. (6) has been utilized)} \tag{11}$$

Note that since $SG_{\text{anchor}} > 1$, $\Delta D < 0$, i.e. the free surface moves downward.

Combine the expressions for $\Delta h$ and $\Delta D$ to determine the movement of the raft bottom relative to the pit walls.

$$D + H - h = \left(D + \Delta D\right) + \left(H + \Delta H\right) - \left(h + \Delta h\right) \tag{12}$$

$$\Delta H = -\Delta D + \Delta h \tag{13}$$

$$\Delta H = -\frac{\left(1 - SG_{\text{anchor}}\right) V_{\text{anchor}}}{L^2} - \frac{SG_{\text{anchor}} V_{\text{anchor}}}{l^2} \tag{14}$$

$$\boxed{\therefore \Delta H = \frac{V_{\text{anchor}}}{L^2}\left[SG_{\text{anchor}}\left(1 - \frac{L^2}{l^2}\right) - 1\right]} \tag{15}$$

Use the given data to determine $\Delta H$.

$V_{\text{anchor}} = 1 \text{ ft}^3$
$L = 4$ ft
$SG_{\text{anchor}} = 7.85$
$l = 3$ ft
$\Rightarrow$ $\boxed{\Delta H = -0.44 \text{ ft (The raft moves closer to the top of the pit.)}}$

Recall that $H = 7.5$ ft and Bond can only reach $R = 7$ ft. After tossing the anchor overboard, the bottom of the raft is $H + \Delta H = 7.06 \text{ ft} > R = 7$ ft. $\boxed{\text{Hence, Bond still can't reach the top of the pit.}}$

Goodbye, Mr. Bond.