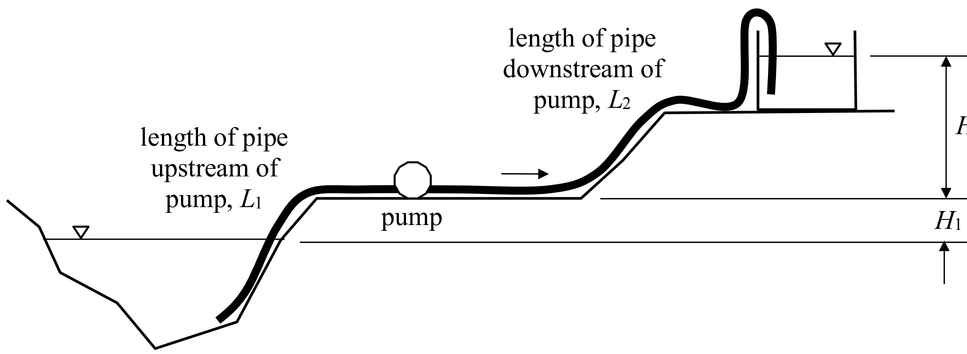
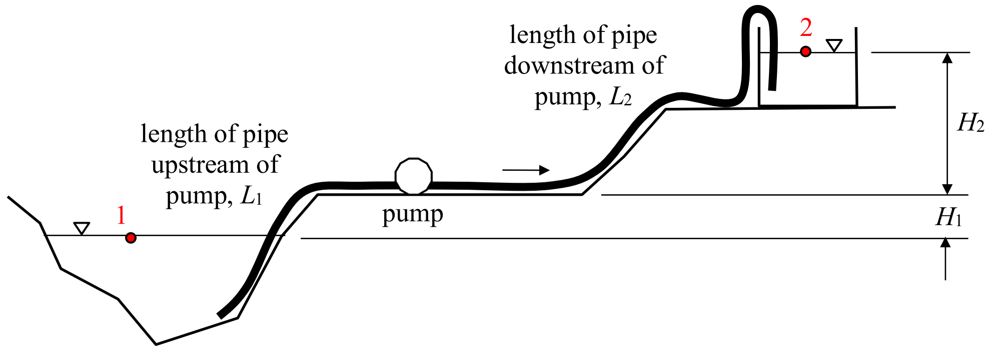
Consider the pipe system containing a pump shown in the figure below. The fluid being pumped from the lake to the tank is water (density = 1000 kg/m^3, kinematic viscosity = 1.0*10^{-6} m^2/s).



diameter of both lengths of pipe, $D_1 = D_2 = 10$ cm
length of pipe upstream of pump, $L_1 = 5$ m
length of pipe downstream of pump, $L_2 = 15$ m
roughness of both lengths of pipe, $\varepsilon_1 = \varepsilon_2 = 1.5*10^{-4}$ m
total minor loss upstream of pump, $K_{\text{minor},1} = 1.0$
total minor loss downstream of pump, $K_{\text{minor},2} = 2.0$
$H_1 = 3$ m
$H_2 = 10$ m
pump head rise curve: $H\,[\text{m}] = (-1.5*10^3\ \text{s}^2/\text{m}^5)Q^2 + (2.8*10^1\ \text{s/m}^2)Q + (6.3*10^1\ \text{m})$
pump efficiency curve: $\eta = (-5.6*10^1\ \text{s}^2/\text{m}^6)Q^2 + (1.2*10^1\ \text{s/m}^3)Q + (2.1*10^{-1})$

a. Determine the operating flow rate for the system.
b. What power must be supplied to the pump by the motor to operate at the flow rate found in part (a)?

SOLUTION:
To determine the operating flow rate, first determine the system head curve by applying the extended Bernoulli equation from point 1 to point 2.



$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_L+H_S\ , \tag{1}$$

where

$p_1 = p_{atm}$	$p_2 = p_{atm}$
$\bar{V}_1 \approx 0$	$\bar{V}_2 \approx 0$
$z_1 = -H_1$	$z_2 = H_2$

$$H_L=\sum_i K_i\frac{\bar{V}_i^2}{2g}$$
$$=\left[f\left(\frac{L_1+L_2}{D}\right)+K_{\text{minor},1}+K_{\text{minor},2}\right]\frac{\bar{V}_p^2}{2g}=\left[f\left(\frac{L_1+L_2}{D}\right)+K_{\text{minor},1}+K_{\text{minor},2}\right]\frac{Q^2}{2g\left(\frac{\pi}{4}D^2\right)^2}\ . \tag{2}$$

The friction factor may be found from the Moody diagram. Since the flow rate is unknown, try assuming that the flow is in the fully turbulent region of the Moody diagram (this assumption will need to be verified). In this region, the friction factor is only a function of the pipe's relative roughness,

$$\frac{\varepsilon}{D}=\frac{1.5*10^{-4}\text{ m}}{0.10\text{ m}}=1.5*10^{-3} \tag{3}$$

From the Moody diagram, $f = 0.022$.

Combining these relations gives the system head curve,

$$H_{S,\text{system}}=\left(z_2-z_1\right)+\left[f\left(\frac{L_1+L_2}{D}\right)+K_{\text{minor},1}+K_{\text{minor},2}\right]\frac{Q^2}{2g\left(\frac{\pi}{4}D^2\right)^2}=s_0+s_2Q^2\ , \tag{4}$$

where,

$$s_0=z_2-z_1\ \text{, and} \tag{5}$$

$$s_2=\frac{1}{2g\left(\frac{\pi}{4}D^2\right)^2}\left[f\left(\frac{L_1+L_2}{D}\right)+K_{\text{minor},1}+K_{\text{minor},2}\right]\ . \tag{6}$$

Determine the operating point by equating the system head curve to the pump head curve,

$$s_0+s_2Q^2=p_0+p_1Q+p_2Q^2\ , \tag{7}$$

$$\left(p_2-s_2\right)Q^2+p_1Q+\left(p_0-s_0\right)=0\ , \tag{8}$$

$$Q=\frac{-p_1\pm\sqrt{p_1^2-4\left(p_2-s_2\right)\left(p_0-s_0\right)}}{2\left(p_2-s_2\right)}\ . \tag{9}$$

Using the given data,

s_0 = 13 m
s_2 = $6.08*10^3$ s^2/m^5
p_0 = $6.30*10^1$ m
p_1 = $2.80*10^1$ s/m^2
p_2 = $-1.50*10^3$ s^2/m^5

$\boxed{Q = 8.3*10^{-2}\ \text{m}^3/\text{s}}$

Check the Reynolds number assumption of fully turbulent flow,

$$\bar{V} = \frac{Q}{\frac{\pi}{4}D^2} \quad \Rightarrow \quad \bar{V} = 10.6\ \text{m/s} \ , \tag{10}$$

$$\text{Re} = \frac{\bar{V}D}{\nu} \quad \Rightarrow \text{Re} = 1.1*10^6 \tag{11}$$

Checking the Moody diagram shows that the flow is in the fully rough zone for this Reynolds number and relative roughness. Thus, our assumption of fully rough flow was a good one.

The power input to the fluid by the pump at these conditions is,

$$\dot{W}_{\substack{\text{into}\\\text{fluid}}} = \rho Q g H \ , \tag{12}$$

where H = 55.0 m at the operating flow rate of $8.3*10^{-2}$ m^3/s (found using either the system or pump head curves). Hence,

$$\Rightarrow \ \dot{W}_{\substack{\text{into}\\\text{fluid}}} = 44.8 \text{ kW} \ . \tag{13}$$

Since the pump isn't 100% efficient, the power that must be supplied to the pump is,

$$\dot{W}_{\substack{\text{into}\\\text{pump}}} = \frac{\dot{W}_{\substack{\text{into}\\\text{fluid}}}}{\eta} \quad \Rightarrow \boxed{\dot{W}_{\substack{\text{into}\\\text{pump}}} = 54.6 \text{ kW}}. \tag{14}$$

where the pump efficiency at the operating point is η = 82% (using the given efficiency curve for the pump, $\eta = (-5.6*10^1\ s^2/m^6)Q^2 + (1.2*10^1\ s/m^3)Q + (2.1*10^{-1})$).