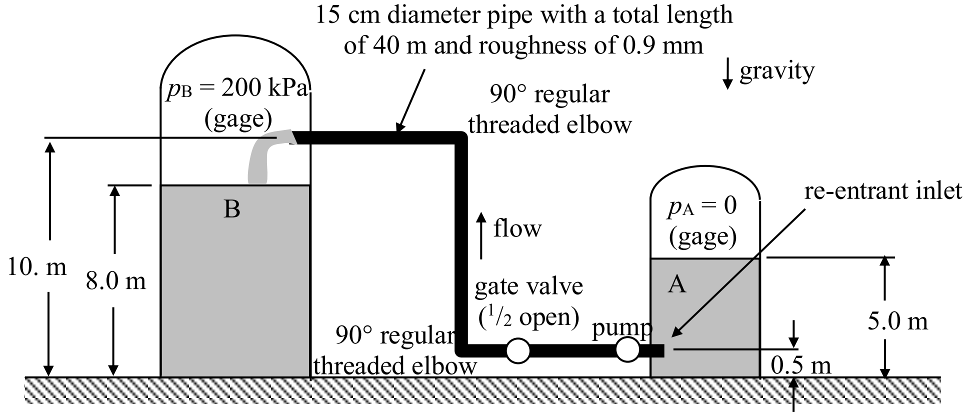
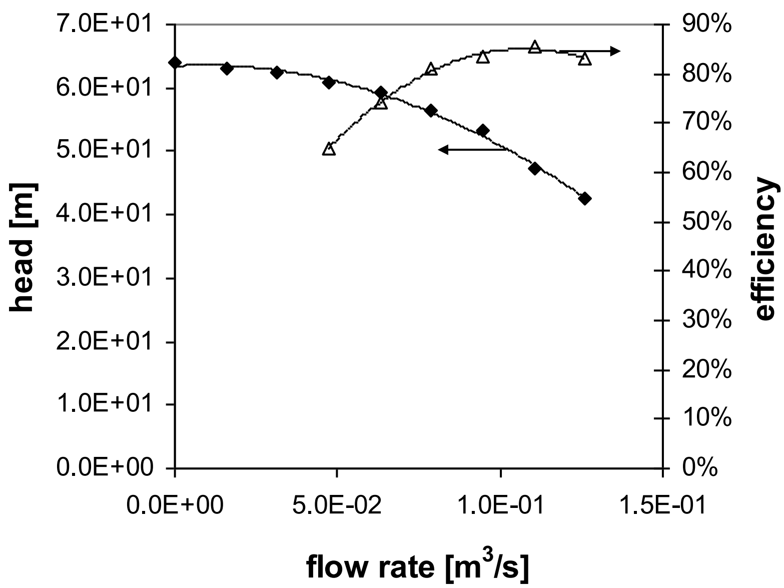
Consider the pipe/pump system shown below in which water (with a density of 1.0E3 kg/m^3 and dynamic viscosity of 1.3E-3 Pa·s) is pumped from tank A to tank B.



The pump to be used in the system has the following pump performance curve.



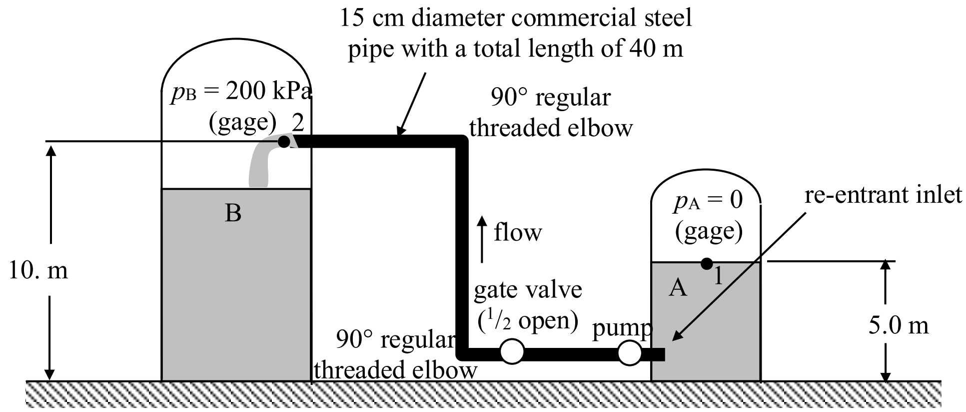
Curve fits to the pump performance data are given below:

H [m] = (-1.5E3 s^2/m^5) Q^2 + (2.8E1 s/m^2) Q + (6.3E1 m)

η_P = (-5.6E1 s^2/m^6) Q^2 + (1.2E1 s/m^3) Q + (2.1E-1)

a. Determine the operating point for the system.
b. Is the given pump efficient for this application? Explain your answer.
c. Do you anticipate that cavitation in the pump will be an issue? Explain your answer.

SOLUTION:



Apply the Extended Bernoulli Equation from the free surface of tank A (point 1) to the end of the pipe leading into tank B (point 2).

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_{L,12}+H_{S,12} \quad (1)$$

where

$p_1 = p_A = 0$ (gage) and $p_2 = p_B = 2.0\text{E}5$ Pa (gage) (given)

$\bar{V}_1 \approx 0$ (large tank)

$\bar{V}_2 = \bar{V}_P = \dfrac{Q}{\frac{\pi}{4}D^2}$ and $\alpha_2 \approx 1$ (assuming turbulent flow)

$z_1 = 5.0\text{E}0$ m and $z_2 = 1.0\text{E}1$ m (given)

$$H_{L,12}=f\left(\frac{L}{D}\right)\frac{\bar{V}_P^2}{2g}+K_{\substack{\text{re-entrant}\\\text{inlet}}}\frac{\bar{V}_P^2}{2g}+K_{\substack{\text{1/2 open}\\\text{gate valve}}}\frac{\bar{V}_P^2}{2g}+2K_{90\ \substack{\text{threaded}\\\text{elbow}}}\frac{\bar{V}_P^2}{2g}$$

$$H_{L,12}=\left[f\left(\frac{L}{D}\right)+K_{\substack{\text{re-entrant}\\\text{inlet}}}+K_{\substack{\text{1/2 open}\\\text{gate valve}}}+2K_{90\ \substack{\text{threaded}\\\text{elbow}}}\right]\frac{\bar{V}_P^2}{2g} \quad (2)$$

(Note that there are no exit losses at point 2.)

The friction factor, f, is determined from the Moody chart using the Reynolds number in the pipe, Re, and the relative roughness, ε/D. Since the Reynolds number is unknown at this point (since the flow rate and hence velocity are unknown), assume that the flow occurs in the fully rough zone. The pipe has a roughness of 0.9 mm. Hence:

$$\frac{\varepsilon}{D} = \frac{(9.0\text{E-4 m})}{(1.5\text{E-1 m})} = 6.0\text{E-3}$$

$$f = 3.2\text{E-2}$$

Hence, the major loss coefficient for the system is:

$$K_{\text{major}} = f\left(\frac{L}{D}\right) = (3.2\text{E-2})\left(\frac{4.0\text{E1 m}}{1.5\text{E-1 m}}\right) = 8.6\text{E0}$$

The minor loss coefficients are found from minor loss tables to be:

$$K_{\substack{\text{re-entrant}\\ \text{inlet}}} = 8.0\text{E-1}$$

$$K_{\substack{\text{half open}\\ \text{gate valve}}} = 2.1\text{E0}$$

$$K_{\substack{90\ \text{threaded}\\ \text{elbow}}} = 1.5\text{E0}$$

Re-arrange Eqn. (1) to solve for $H_{S,12}$ and substitute the values given above.

$$H_{S,12} = \frac{p_2 - p_1}{\rho g} + \alpha_2 \frac{\bar{V}_2^2}{2g} - \alpha_1 \frac{\bar{V}_1^2}{2g} + z_2 - z_1 + H_{L,12}$$

$$= \frac{p_2}{\rho g} + \frac{Q^2}{2g\left(\frac{\pi}{4}D^2\right)^2} + z_2 - z_1 + H_{L,12}$$

$$= \frac{(2.0\text{E5 Pa})}{(1.0\text{E3 kg/m}^3)(9.8\text{E0 m/s}^2)} + \frac{Q^2}{2(9.8\text{E0 m/s}^2)\left[\frac{\pi}{4}(1.5\text{E-1 m})^2\right]^2} + (1.0\text{E1 m}) - (5.0\text{E0 m})$$

$$+\left[8.6\text{E0} + 8.0\text{E-1} + 2.1\text{E0} + 2(1.5\text{E0})\right]\frac{Q^2}{2(9.8\text{E0 m/s}^2)\left[\frac{\pi}{4}(1.5\text{E-1 m})^2\right]^2}$$

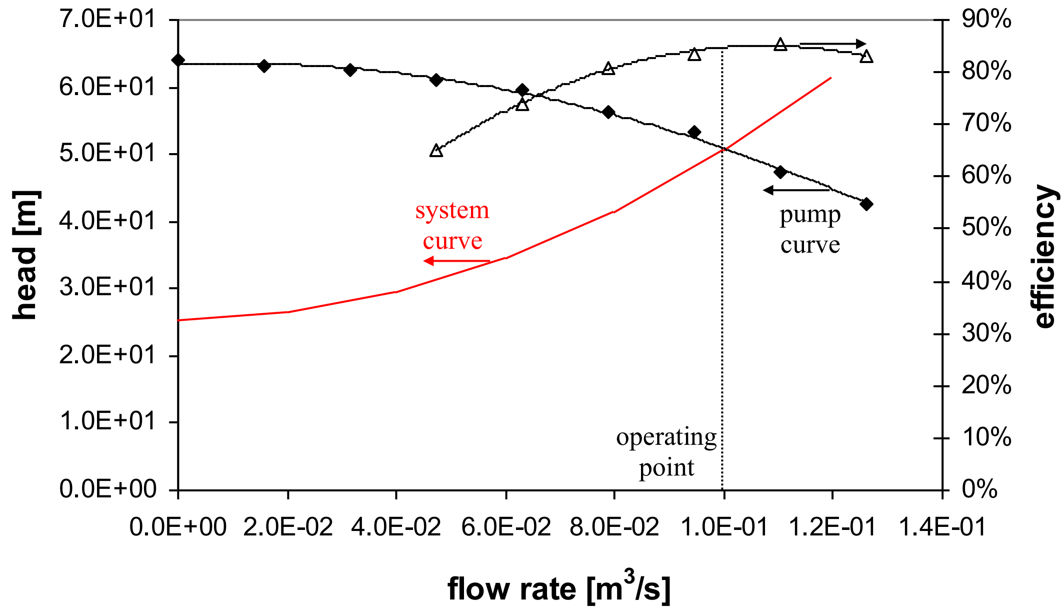
$$H_{S,12} = (2.5\text{E1 m}) + (2.5\text{E3 s}^2/\text{m}^5)Q^2$$

Equate the system head curve with the given pump head curve to determine the operating point.

$$\underbrace{(2.5\text{E1 m}) + (2.5\text{E3 s}^2/\text{m}^5)Q^2}_{\text{system curve}} = \underbrace{(6.3\text{E1 m}) + (2.8\text{E1 s/m}^2)Q + (\text{-}1.5\text{E3 s}^2/\text{m}^5)Q^2}_{\text{pump curve}}$$

$$(4.0\text{E3 s}^2/\text{m}^5)Q^2 + (\text{-}2.8\text{E1 s/m}^2)Q + (\text{-}3.8\text{E1 m}) = 0$$

$$\boxed{Q = 1.0\text{E-1 m}^3/\text{s}}$$



The velocity corresponding to this flow rate is:

$$\bar{V}_2 = \frac{Q}{\frac{\pi}{4}D^2} = 5.7\text{E}0 \text{ m/s}$$

and the corresponding Reynolds number is:

$$\text{Re} = \frac{\bar{V}_2 D}{\nu} = 6.5\text{E}5$$

Hence, the assumption of fully rough turbulent flow is ok.

The pump efficiency at this flow rate is found using the given pump efficiency curve.

$$\eta_P = \left(\text{-5.6E1 s}^2/\text{m}^6\right)Q^2 + \left(\text{1.2E1 s/m}^3\right)Q + \left(\text{2.1E-1}\right)$$

$$\eta_P = 85\%$$

Since this efficiency is very close to the best efficiency point, this is an efficient pump for this application.

To determine if cavitation will occur in the pump, we would need to compare the NPSH available at the pump inlet to the NPSH required by the pump (need NPSHA > NPSHR to avoid cavitation). The NPSHA can be determined by apply the Extended Bernoulli Equation from point 1 to a point at the inlet of the pump and using the definition of NPSH.

$$\left(\frac{p}{\rho g} + \alpha\frac{\bar{V}^2}{2g} + z\right)_{\text{inlet}} = \left(\frac{p}{\rho g} + \alpha\frac{\bar{V}^2}{2g} + z\right)_1 - H_{L,1-\text{inlet}} + H_{S,1-\text{inlet}}$$

$$NPSH \equiv \left(\frac{p}{\rho g} + \frac{\bar{V}^2}{2g}\right)_{\text{inlet}} - \frac{p_v}{\rho g}$$

where

$$\alpha_{\text{inlet}} \approx 1 \qquad p_1 = p_{\text{atm}} \qquad H_{S,1-\text{inlet}} = 0$$

$$NPSHA = \frac{p_{\text{atm}} - p_v}{\rho g} + z_1 - z_2 - H_{L,1-\text{inlet}}$$

Since $p_{atm} > p_v$, $z_1 > z_2$, and $H_{L,1\text{-inlet}}$ will be relatively small since there are few loss mechanisms occurring upstream of the pump, cavitation in the pump will most likely not be an issue.