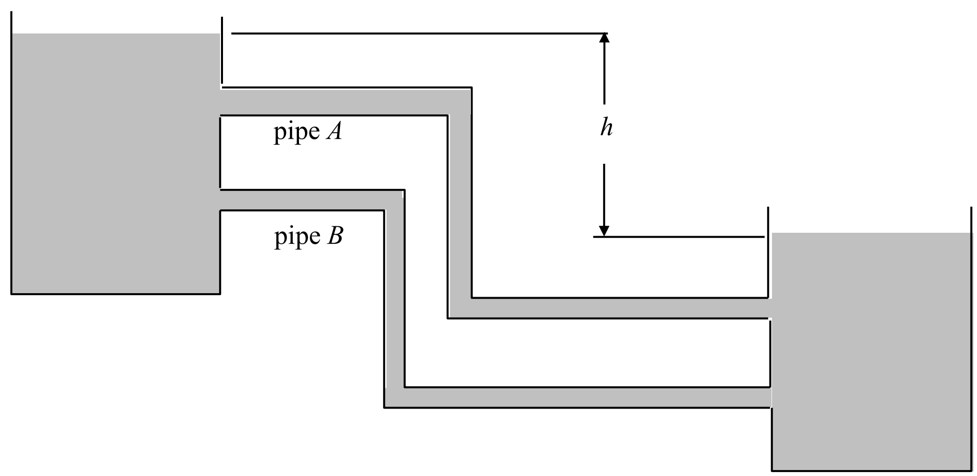
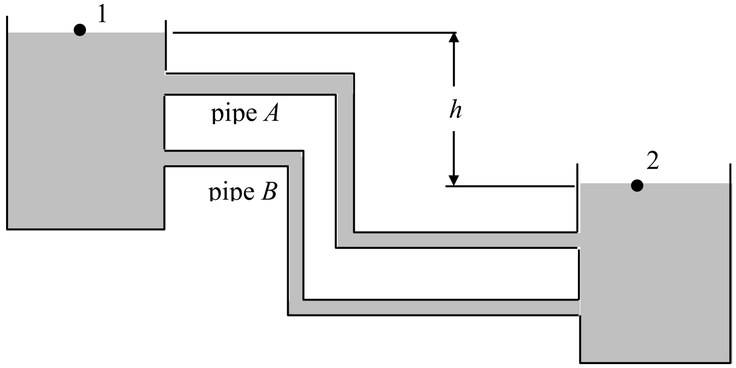
Two water reservoirs are connected by galvanized iron pipes. Assume $D_A = 75$ mm, $D_B = 50$ mm, and $h = 10.5$ m. The length of both pipes is 100 m. Compare the head losses in pipes *A* and *B*. Compute the volume flow rate in each pipe.



SOLUTION:

Apply the Extended Bernoulli Equation from point 1 to point 2 traveling through each pipe.



$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_{L,12}+H_{S,12}$$

where

$p_1=p_2=p_{\text{atm}}$ (free surfaces)

$\bar{V}_1\approx\bar{V}_2\approx 0$ (surface of large tanks)

$z_1-z_2=h$ (given)

$H_{S,12}=0$ (no fluid machines between points 1 and 2)

$$H_{L,12,A}=K_{\text{major},A}\frac{\bar{V}_A^2}{2g}+K_{\text{entrance}}\frac{\bar{V}_A^2}{2g}+2K_{\text{elbow}}\frac{\bar{V}_A^2}{2g}+K_{\text{exit}}\frac{\bar{V}_A^2}{2g}$$

$$H_{L,12,B}=K_{\text{major},B}\frac{\bar{V}_B^2}{2g}+K_{\text{entrance}}\frac{\bar{V}_B^2}{2g}+2K_{\text{elbow}}\frac{\bar{V}_B^2}{2g}+K_{\text{exit}}\frac{\bar{V}_B^2}{2g}$$

and

$$K_{\text{major},A}=f_A\left(\frac{L_A}{D_A}\right)\quad\text{and}\quad K_{\text{major},B}=f_B\left(\frac{L_B}{D_B}\right)$$

Substituting into the Extended Bernoulli Equation gives:

$$h=\left[f_A\left(\frac{L_A}{D_A}\right)+K_{\text{entrance}}+2K_{\text{elbow}}+K_{\text{exit}}\right]\frac{\bar{V}_A^2}{2g} \tag{1}$$

$$h=\left[f_B\left(\frac{L_B}{D_B}\right)+K_{\text{entrance}}+2K_{\text{elbow}}+K_{\text{exit}}\right]\frac{\bar{V}_B^2}{2g} \tag{2}$$

From Eqns. (1) and (2) we observe that $\boxed{\text{the head loss in each pipe is the same and equal to 10.5 m}}$.

The pipes are made of galvanized iron so the roughness of the pipes is ε = 0.15 mm (found from an average roughness table). Hence, the relative roughness in each pipe is:

$$\left.\frac{\varepsilon}{D}\right|_A=\frac{0.15\text{ mm}}{75\text{ mm}}=0.0020\quad\text{and}\quad\left.\frac{\varepsilon}{D}\right|_B=\frac{0.15\text{ mm}}{50\text{ mm}}=0.0030$$

Since we don't yet know the velocity in each pipe, assume that the flows are in the wholly turbulent flow region so that the friction factor is independent of the Reynolds number. For this case, the Moody chart (or the Colebrook relation) indicates that the friction factors corresponding to the relative roughnesses determined above are:

$$f_A = 0.0234 \quad \text{and} \quad f_B = 0.0262$$

Substitute the given data into Eqns. (1) and (2).

h	=	10.5 m
K_{entrance}	=	0.5 (assuming a sharp-edged entrance)
K_{elbow}	=	1.5 (assuming 90° threaded elbows)
K_{exit}	=	1.0
g	=	9.81 m/s^2
L_A	=	100 m
L_B	=	100 m
D_A	=	75 mm = 7.5*10^{-2} m
D_B	=	50 mm = 5.0*10^{-2} m
f_A	=	0.0234 (from above)
f_B	=	0.0262 (from above)
ν_{H20}	=	1.01*10^{-6} m^2/s (water at 20 °C)

Note that the sum of the minor loss coefficients (= 4.5 = 0.5 + 2*1.5 + 1) are not insignificant compared to the major loss coefficients so the minor losses cannot be neglected without significant error.

$$K_{\text{major},A} = 31.2 \quad \text{and} \quad K_{\text{major,B}} = 52.4$$

Solving for the average velocities gives:

$$\bar{V}_A = 2.40 \text{ m/s} \quad \text{and} \quad \bar{V}_B = 1.90 \text{ m/s}$$

The corresponding Reynolds numbers are:

$$\text{Re}_A = \frac{\bar{V}_A D_A}{\nu_{\text{H20}}} = 1.78*10^5 \quad \text{and} \quad \text{Re}_B = \frac{\bar{V}_B D_B}{\nu_{\text{H20}}} = 9.42*10^4 \tag{3}$$

Unfortunately, these Reynolds numbers do not put us in the wholly turbulent zone on the Moody chart (although it's very close) so we must try iterating to a solution instead. For a new choice of friction factors, use the Reynolds number given in Eqn. (3) and consult the Moody chart (or the Colebrook formula).

$$f_A = 0.0244 \quad \text{and} \quad f_B = 0.0275$$

Using these friction factors we find:

$$\bar{V}_A = 2.36 \text{ m/s} \quad \text{and} \quad \bar{V}_B = 1.86 \text{ m/s}$$

and

$$\text{Re}_A = \frac{\bar{V}_A D_A}{\nu_{\text{H20}}} = 1.75*10^5 \quad \text{and} \quad \text{Re}_B = \frac{\bar{V}_B D_B}{\nu_{\text{H20}}} = 9.21*10^4$$

Fortunately these Reynolds numbers give the same friction factors that we started with.

Thus, the volumetric flow rate through each pipe is:

$$\boxed{Q_A = \bar{V}_A \frac{\pi D_A^2}{4} = 1.04*10^{-2} \text{ m}^3/\text{s}}$$

$$\boxed{Q_B = \bar{V}_B \frac{\pi D_B^2}{4} = 3.65*10^{-3} \text{ m}^3/\text{s}}$$