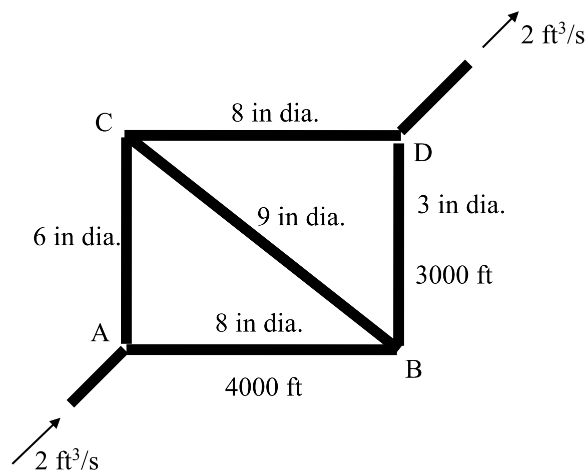
In the five-pipe horizontal network shown in the figure, assume that all pipes have a friction factor f=0.025. For the given inlet and exit flow rate of 2 ft^3/s of water at 20 °C, determine the flow rate and direction in all pipes. If p_A=120 lb_f/in^2 (gage), determine the pressures at points B, C, and D.



SOLUTION:

Apply the Extended Bernoulli Equation around the loops ABCA and DBCD.

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_A=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_A-H_{L,ABCA}+H_{S,ABCA} \quad (1)$$

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_D=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_D-H_{L,DBCD}+H_{S,DBCD} \quad (2)$$

Note that the shaft head terms ($H_{S,ABCA}$ and $H_{S,DBCD}$) are zero since there are no fluid machines in the loops. Simplifying Eqns. (1) and (2) gives:

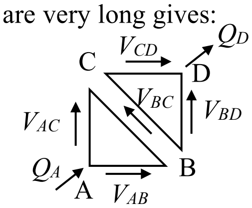
$$H_{L,ABCA}=0 \quad (3)$$

$$H_{L,DBCD}=0 \quad (4)$$

Expanding the head loss term and neglecting minor losses since the pipes are very long gives:

$$H_{L,ABCA}=f_{AB}\left(\frac{L_{AB}}{D_{AB}}\right)\frac{\bar{V}_{AB}^2}{2g}+f_{BC}\left(\frac{L_{BC}}{D_{BC}}\right)\frac{\bar{V}_{BC}^2}{2g}-f_{AC}\left(\frac{L_{AC}}{D_{AC}}\right)\frac{\bar{V}_{AC}^2}{2g}=0 \quad (5)$$

$$H_{L,DBCD}=-f_{BD}\left(\frac{L_{BD}}{D_{BD}}\right)\frac{\bar{V}_{BD}^2}{2g}+f_{BC}\left(\frac{L_{BC}}{D_{BC}}\right)\frac{\bar{V}_{BC}^2}{2g}+f_{CD}\left(\frac{L_{CD}}{D_{CD}}\right)\frac{\bar{V}_{CD}^2}{2g}=0 \quad (6)$$



Note that particular velocity directions have been assumed in the head loss expressions.

At each pipe node the volumetric flow rate must be conserved (conservation of mass). Hence:

at node A: $Q_A=Q_{AB}+Q_{AC} \quad\Rightarrow\quad Q_A=\bar{V}_{AB}\frac{\pi D_{AB}^2}{4}+\bar{V}_{AC}\frac{\pi D_{AC}^2}{4}$ (7)

at node B: $Q_{AB}=Q_{BC}+Q_{BD} \quad\Rightarrow\quad \bar{V}_{AB}\frac{\pi D_{AB}^2}{4}=\bar{V}_{BC}\frac{\pi D_{BC}^2}{4}+\bar{V}_{BD}\frac{\pi D_{BD}^2}{4}$ (8)

at node C: $Q_{CD}=Q_{AC}+Q_{BC} \quad\Rightarrow\quad \bar{V}_{CD}\frac{\pi D_{CD}^2}{4}=\bar{V}_{AC}\frac{\pi D_{AC}^2}{4}+\bar{V}_{BC}\frac{\pi D_{BC}^2}{4}$ (9)

at node D: $Q_D=Q_{BD}+Q_{CD} \quad\Rightarrow\quad Q_D=\bar{V}_{BD}\frac{\pi D_{BD}^2}{4}+\bar{V}_{CD}\frac{\pi D_{CD}^2}{4}$ (10)

Simplify and summarize Eqns. (5) - (10).

$$f_{AB}\left(\frac{L_{AB}}{D_{AB}}\right)\bar{V}_{AB}^2 + f_{BC}\left(\frac{L_{BC}}{D_{BC}}\right)\bar{V}_{BC}^2 - f_{AC}\left(\frac{L_{AC}}{D_{AC}}\right)\bar{V}_{AC}^2 = 0 \quad (11)$$

$$-f_{BD}\left(\frac{L_{BD}}{D_{BD}}\right)\bar{V}_{BD}^2 + f_{BC}\left(\frac{L_{BC}}{D_{BC}}\right)\bar{V}_{BC}^2 + f_{CD}\left(\frac{L_{CD}}{D_{CD}}\right)\bar{V}_{CD}^2 = 0 \quad (12)$$

$$\left(\frac{\pi D_{AB}^2}{4}\right)\bar{V}_{AB} + \left(\frac{\pi D_{AC}^2}{4}\right)\bar{V}_{AC} = Q_A \quad (13)$$

$$\left(\frac{\pi D_{BC}^2}{4}\right)\bar{V}_{BC} + \left(\frac{\pi D_{BD}^2}{4}\right)\bar{V}_{BD} - \left(\frac{\pi D_{AB}^2}{4}\right)\bar{V}_{AB} = 0 \quad (14)$$

$$\left(\frac{\pi D_{AC}^2}{4}\right)\bar{V}_{AC} + \left(\frac{\pi D_{BC}^2}{4}\right)\bar{V}_{BC} - \left(\frac{\pi D_{CD}^2}{4}\right)\bar{V}_{CD} = 0 \quad (15)$$

$$\left(\frac{\pi D_{BD}^2}{4}\right)\bar{V}_{BD} + \left(\frac{\pi D_{CD}^2}{4}\right)\bar{V}_{CD} = Q_D \quad (16)$$

Note that Eqn. (16) is not independent since it can be formed by adding Eqns. (13) and (14), subtracting Eqn. (15) and noting that $Q_D = Q_A$. Hence, Eqns. (11) - (15) represent five equations with five unknowns ($\bar{V}_{AB}, \bar{V}_{BC}, \bar{V}_{AC}, \bar{V}_{BD}$, and $\bar{V}_{CD}$). Note that f_{AB}, f_{BC}, f_{AC}, f_{BD}, and f_{CD} are given in the problem statement along with each pipe's length and diameter and the volumetric flowrate Q_A.

Using the given data:

$f_{\text{all pipes}}$	=	0.025
$L_{AB} = L_{CD}$	=	4000 ft
$L_{AC} = L_{BD}$	=	3000 ft
L_{BC}	=	5000 ft (from the Pythagorean theorem)
$D_{AB} = D_{CD}$	=	8/12 ft
D_{AC}	=	6/12 ft
D_{BD}	=	3/12 ft
D_{BC}	=	9/12 ft
Q_A	=	2 ft^3/s

The system of non-linear algebraic equations (Eqns. (11) - (15)) can be solved iteratively. One approach is given below.

1. Assume a value of $\bar{V}_{AB}$.
2. Solve for $\bar{V}_{AC}$ using Eqn. (13).
3. Solve for $\bar{V}_{BC}$ using Eqn. (11).
4. Solve for $\bar{V}_{CD}$ using Eqn. (15).
5. Solve for $\bar{V}_{BD}$ using Eqn. (12).
6. Solve for $\bar{V}_{AB}$ using Eqn. (14).
7. Are the $\bar{V}_{AB}$ s from step 6 and step 1 equal? If so, then the iterations are finished. If not, then choose a new value for $\bar{V}_{AB}$ and go to step 2.

After some iteration.

$\bar{V}_{AB}$	=	$3.40*10^{0}$ ft/s	⇒	Q_{AB}	=	$1.19*10^{0}$ ft^3/s
$\bar{V}_{AC}$	=	$4.14*10^{0}$ ft/s	⇒	Q_{AC}	=	$8.13*10^{-1}$ ft^3/s
$\bar{V}_{BC}$	=	$2.24*10^{0}$ ft/s	⇒	Q_{BC}	=	$9.90*10^{-1}$ ft^3/s
$\bar{V}_{CD}$	=	$5.17*10^{0}$ ft/s	⇒	Q_{CD}	=	$1.80*10^{0}$ ft^3/s
$\bar{V}_{BD}$	=	$4.02*10^{0}$ ft/s	⇒	Q_{BD}	=	$1.97*10^{-1}$ ft^3/s

To find the pressure at the various nodes, apply the Extended Bernoulli Equation between the nodes.

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_B=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_A-H_{L,AB}+H_{S,AB}$$

where

$p_A = 120$ psig

$p_B = ?$

$V_A = V_B$ (the velocity just upstream of point B is equal to the velocity just downstream of point A)

$z_A = z_B$

$H_{S,AB} = 0$

$\rho_{H20\ @\ 20\ °C} = 1.94$ slug/ft^3 (Note: 1 lb$_f$ = 1 slug·ft/s^2)

$$H_{L,AB}=f_{AB}\left(\frac{L_{AB}}{D_{AB}}\right)\frac{\bar{V}_{AB}^2}{2g}$$

$$\Rightarrow\ p_B=p_A-\tfrac{1}{2}\rho\bar{V}_{AB}^2 f_{AB}\left(\frac{L_{AB}}{D_{AB}}\right)$$

$p_B = 1.08*10^{2}$ psig

Using a similar approach from A to C (or from B to C):

$p_C = 1.03*10^{2}$ psig

Using a similar approach from B to D (or from C to D):

$p_D = 7.57*10^{1}$ psig