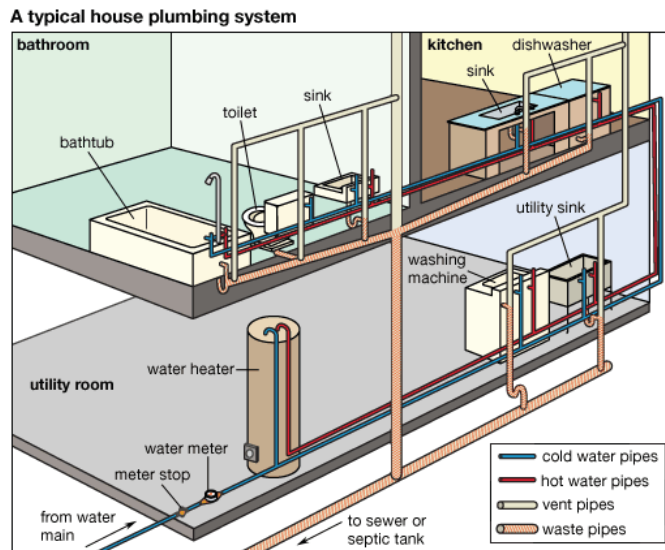


A typical home plumbing system is shown in the following figure. We wish to analyze the hot water supply system from the water heater to the bathtub.



Between the water heater outlet and exit of the bathtub faucet there is:

- 30.5 m length of  $1.38 \times 10^{-2}$  m inner diameter copper pipe (roughness of  $1.52 \times 10^{-6}$  m)
- 3.05 m of elevation gain
- One re-entrant inlet
- Six regular, flanged 90 deg. pipe elbows
- Five line flow, flanged tee couplings
- One fully open disc valve bathtub faucet with a loss coefficient of 11. The exit diameter of the faucet is the same as diameter of the pipe.

Assume the bathtub faucet is open, but the valves to each of the other appliances and sinks are closed (i.e., there is only flow through the bathtub faucet). For these conditions, the volumetric flow rate through the faucet is  $3.79 \times 10^{-4}$  m<sup>3</sup>/s. For water at a temperature of 60 °C, the density is 983 kg/m<sup>3</sup> and the dynamic viscosity is  $4.67 \times 10^{-4}$  Pa.s.

- Calculate the pressure drop (in Pascals) across the bathtub faucet.
- Determine the total minor loss (in meters) between the water heater outlet and the exit of the bathtub faucet.
- Calculate the friction factor for the pipe.
- Determine the total major loss (in meters) between the water heater outlet and the exit of the bathtub faucet.
- Determine the gage pressure required in the water heater (in Pascals) to provide the given flow rate of water.

SOLUTION:

The pressure drop across the faucet is,

$$\Delta p = K_{\text{faucet}} \left( \frac{1}{2} \rho \bar{V}^2 \right), \quad (1)$$

where

$$\bar{V} = \frac{Q}{\frac{\pi}{4} D^2}. \quad (2)$$

Using the given data,

$$\Rightarrow \bar{V} = 2.52 \text{ m/s} \quad (3)$$

$$\Rightarrow \Delta p_{\text{faucet}} = 3.42 * 10^4 \text{ Pa}$$

The total minor loss is,

$$H_{L,\text{minor}} = (K_{\text{inlet}} + 6K_{\text{elbow}} + 5K_{\text{tee}} + K_{\text{faucet}}) \frac{\bar{V}^2}{2g}, \quad (4)$$

where

$$K_{\text{inlet}} = 0.8 \text{ (from the minor loss table)}$$

$$K_{\text{elbow}} = 0.3 \text{ (from the minor loss table)}$$

$$K_{\text{tee}} = 0.2 \text{ (from the minor loss table)}$$

$$\Rightarrow H_{L,\text{minor}} = 4.71 \text{ m}$$

The friction factor is found from the Moody chart to be,

$$f = 0.019,$$

given that the Reynolds number is,

$$\text{Re}_D = \frac{\rho \bar{V} D}{\mu} = 7.33 * 10^4, \quad (5)$$

and the relative roughness is,

$$\frac{\epsilon}{D} = 1.10 * 10^{-4}. \quad (6)$$

The total major loss is,

$$H_{L,\text{major}} = f \left( \frac{L}{D} \right) \frac{\bar{V}^2}{2g} \Rightarrow H_{L,\text{major}} = 13.80 \text{ m}. \quad (7)$$

The pressure in the water heater may be found using the Extended Bernoulli Equation from point 1 (inside the water heater) to point 2 (downstream of the faucet),

$$\left( \frac{p}{\rho g} + \alpha \frac{\bar{V}^2}{2g} + z \right)_2 = \left( \frac{p}{\rho g} + \alpha \frac{\bar{V}^2}{2g} + z \right)_1 - H_L + H_S, \quad (8)$$

where

$$p_1 = p_{\text{water heater (gage)}}$$

$$p_2 = 0 \text{ (gage)}$$

$$\bar{V}_1 = 0 \text{ (large tank)}$$

$$\bar{V}_2 = \bar{V} \text{ (at the exit of the faucet)}$$

$$\alpha \approx 1 \text{ (turbulent flow based on the Reynolds number calculated in Eq. (5))}$$

$$z_2 - z_1 = 3.05 \text{ m}$$

$$H_L = H_{L,\text{minor}} + H_{L,\text{major}} = 18.51 \text{ m}$$

$$H_S = 0 \text{ (no fluid machinery in the system)} \quad (9)$$

Solving for  $p_{\text{water heater}}$  gives,

$$p_{\text{water heater}} = 2.11 * 10^5 \text{ Pa (gage)}.$$

