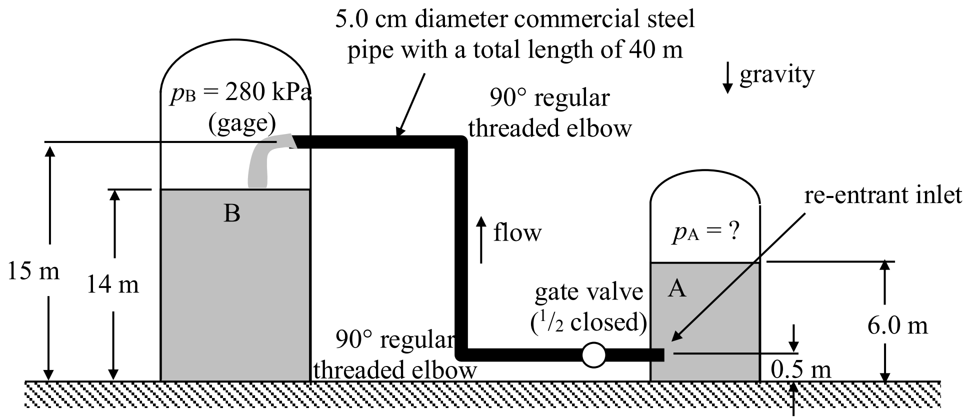
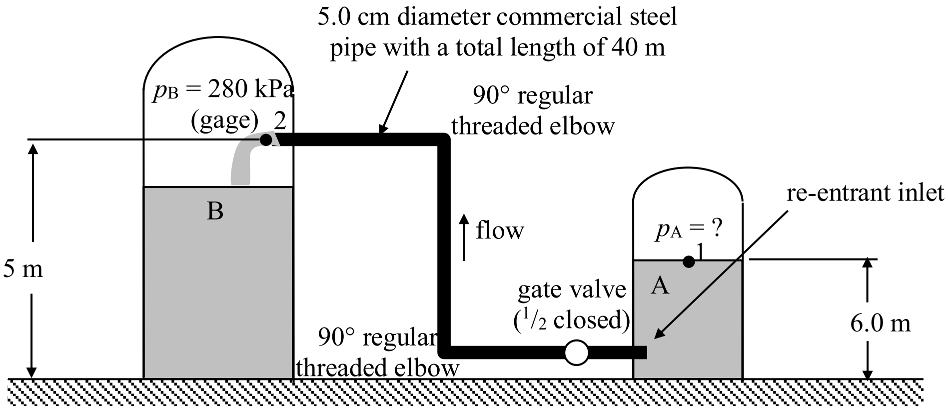
Consider the pipe system shown below in which water (with a density of 1.0E3 kg/m^3 and a dynamic viscosity of 1.3E-3 Pa·s) flows from the tank A to tank B. If the required flow rate is 1.0E-2 m^3/s, what is the required gage pressure in tank A?



SOLUTION:



Apply the Extended Bernoulli Equation from the free surface of tank A (point 1) to the end of the pipe leading into tank B (point 2).

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_{L,12}+H_{S,12} \tag{1}$$

where

$p_1 = p_A = ?$ and $p_2 = p_B = 2.8\text{E}5\text{ Pa (gage)}$ (given)

$\bar{V}_1 \approx 0$ (large tank) $\quad \bar{V}_2 = \bar{V} \quad \alpha_2 \approx 1$ (Assume turbulent flow.)

$z_1 = 6.0\text{E}0\text{ m}$ and $z_2 = 1.5\text{E}1\text{ m}$ (given)

$H_{S,12} = 0$ (no fluid machinery between points 1 and 2)

$$H_{L,12}=f\left(\frac{L}{D}\right)\frac{\bar{V}^2}{2g}+K_{\substack{\text{re-entrant}\\ \text{inlet}}}\frac{\bar{V}^2}{2g}+K_{\substack{\text{1/2 open}\\ \text{gate valve}}}\frac{\bar{V}^2}{2g}+2K_{\substack{90\ \text{threaded}\\ \text{elbow}}}\frac{\bar{V}^2}{2g}$$

$$H_{L,12}=\left[f\left(\frac{L}{D}\right)+K_{\substack{\text{re-entrant}\\ \text{inlet}}}+K_{\substack{\text{1/2 open}\\ \text{gate valve}}}+2K_{\substack{90\ \text{threaded}\\ \text{elbow}}}\right]\frac{\bar{V}^2}{2g} \tag{2}$$

(Note that there are no exit losses at point 2 since no mixing occurs there.)

The mean velocity in the pipe is determined from the volumetric flow rate and the pipe area.

$$\bar{V}=\frac{Q}{\frac{\pi}{4}D^2}$$

Using the given data:

$$\bar{V}=\frac{1.0\text{E-2 m}^3/\text{s}}{\frac{\pi}{4}\left(5.0\text{E-2 m}\right)^2}=5.1\text{E0 m/s}$$

The friction factor, f, is determined from the Moody chart using the Reynolds number in the pipe, Re, and the relative roughness, ε/D.

$$\text{Re}=\frac{\rho\bar{V}D}{\mu}=\frac{\left(1.0\text{E3 kg/m}^3\right)\left(5.1\text{E0 m/s}\right)\left(5.0\text{E-2 m}\right)}{\left(1.3\text{E-3 Pa}\cdot\text{s}\right)}=2.0\text{E5} \quad \text{(Turbulent flow assumption ok.)}$$

$$\frac{\varepsilon}{D}=\frac{\left(4.5\text{E-5 m}\right)}{\left(5.0\text{E-2 m}\right)}=9.0\text{E-4}$$

$$f=2.1\text{E-2}$$

Hence, the major loss coefficient for the system is:

$$K_{\text{major}} = f\left(\frac{L}{D}\right) = (2.1E-2)\left(\frac{4.0\text{E1 m}}{5.0\text{E-2 m}}\right) = 1.7\text{E1}$$

The minor loss coefficients are found from minor loss tables to be:

$$K_{\substack{\text{re-entrant}\\\text{inlet}}} = 8.0\text{E-1}$$

$$K_{\substack{\text{half open}\\\text{gate valve}}} = 2.1\text{E0}$$

$$K_{90\ \substack{\text{threaded}\\\text{elbow}}} = 1.5\text{E0}$$

Using the given data, the total head loss (from Eqn. (2)) is:

$$H_{L,12} = \left[(2.1E-2)\left(\frac{4.0\text{E1 m}}{5.0\text{E-2 m}}\right) + 8.0\text{E-1} + 2.1\text{E0} + 2(1.5\text{E0})\right]\frac{(5.1\text{E0 m/s})^2}{2\left(9.8\text{E0 m/s}^2\right)}$$

$$\therefore H_{L,12} = 3.0\text{E1 m}$$

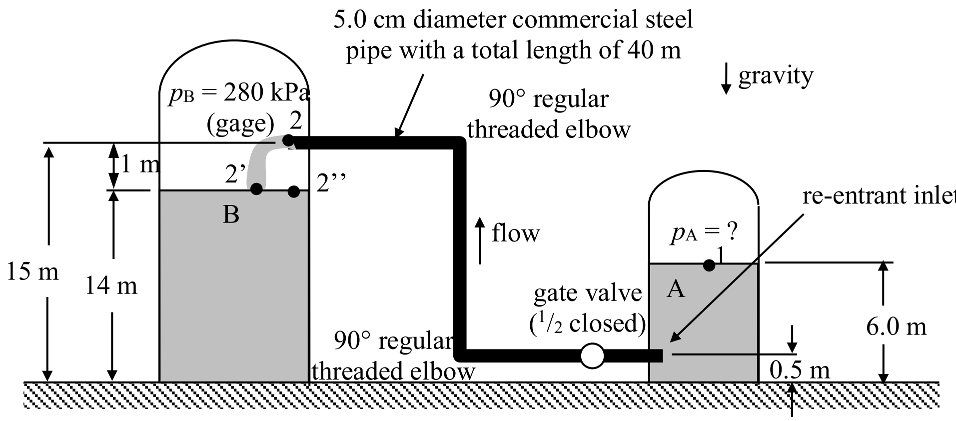
Re-arranging Eqn. (1) to solve for p_1 gives:

$$p_A = p_B + \tfrac{1}{2}\rho\bar{V}^2 + \rho g\left(z_2 - z_1 + H_{L,12}\right)$$

$$= (2.8\text{E5 Pa}) + \tfrac{1}{2}\left(1.0\text{E3 kg/m}^3\right)(9.5\text{E0 m/s})^2 + \left(1.0\text{E3 kg/m}^3\right)\left(9.8\text{E0 m/s}^2\right)(1.5\text{E1 m} - 6.0\text{E0 m} + 3.0\text{E1 m})$$

$$\boxed{p_A = 6.8\text{E5 Pa}}$$

Now let's solve the problem using points 2' and 2'' as shown in the figure below.



Apply the Extended Bernoulli Equation from the free surface of tank A (point 1) to the end of the stream at the surface of the free jet (point 2').

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_{2'}=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_{L,12'}+H_{S,12'} \tag{3}$$

where

$p_1 = p_A = ?$ and $p_{2'} = p_B = 2.8\text{E}5$ Pa (gage) (given)

$\bar{V}_1 \approx 0$ (large tank)

$\bar{V}_{2'}^2 = \bar{V}_2^2 + 2g(1\text{ m})$ (using Bernoulli's Eqn applied from the end of the pipe to the surface of tank B)

$\bar{V}_{2'} = 6.7\text{E}0$ m/s

$\alpha_{2'} \approx 1$ (Assume turbulent flow.)

$z_1 = 6.0\text{E}0$ m and $z_{2'} = 1.4\text{E}1$ m (given)

$H_{S,12'} = 0$ (no fluid machinery between points 1 and 2)

$$H_{L,12'} = f\left(\frac{L}{D}\right)\frac{\bar{V}_2^2}{2g}+K_{\substack{\text{re-entrant}\\\text{inlet}}}\frac{\bar{V}_2^2}{2g}+K_{\substack{\text{1/2 open}\\\text{gate valve}}}\frac{\bar{V}_2^2}{2g}+2K_{\substack{90^\circ\text{ threaded}\\\text{elbow}}}\frac{\bar{V}_2^2}{2g}$$

$$H_{L,12'} = \left[f\left(\frac{L}{D}\right)+K_{\substack{\text{re-entrant}\\\text{inlet}}}+K_{\substack{\text{1/2 open}\\\text{gate valve}}}+2K_{\substack{90^\circ\text{ threaded}\\\text{elbow}}}\right]\frac{\bar{V}_2^2}{2g} \tag{4}$$

(Note that there are no exit losses from point 2 to point 2' since the kinetic energy in the stream hasn't been dissipated.)

Using the same data as in the previous solution, except for the velocity at 2' and the elevation at 2',

$$p_A = p_B + \tfrac{1}{2}\rho\bar{V}_{2'}^2 + \rho g\left(z_{2'} - z_1 + H_{L,12'}\right)$$
$$= (2.8\text{E}5\text{ Pa}) + \tfrac{1}{2}(1.0\text{E}3\text{ kg/m}^3)(6.7\text{E}0\text{ m/s})^2 + (1.0\text{E}3\text{ kg/m}^3)(9.8\text{E}0\text{ m/s}^2)(1.4\text{E}1\text{ m} - 6.0\text{E}0\text{ m} + 3.0\text{E}1\text{ m})$$

$\boxed{p_A = 6.8\text{E}5\text{ Pa}}$ (Same answer as found previously!)

Now apply the Extended Bernoulli Equation from the free surface of tank A (point 1) to the surface of the tank (point 2'').

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_{2''}=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_{1}-H_{L,12''}+H_{S,12''} \tag{5}$$

where

$p_1 = p_A = ?$ and $p_{2'} = p_B = 2.8\text{E}5 \text{ Pa (gage)}$ (given)

$\bar{V}_1 \approx 0$ (large tank)

$\bar{V}_{2''} \approx 0$ (surface of large tank)

$z_1 = 6.0\text{E}0 \text{ m}$ and $z_{2''} = 1.4\text{E}1 \text{ m}$ (given)

$H_{S,12''} = 0$ (no fluid machinery between points 1 and 2)

$$H_{L,12''} = f\left(\frac{L}{D}\right)\frac{\bar{V}_2^2}{2g}+K_{\substack{\text{re-entrant}\\ \text{inlet}}}\frac{\bar{V}_2^2}{2g}+K_{\substack{\text{1/2 open}\\ \text{gate valve}}}\frac{\bar{V}_2^2}{2g}+2K_{\substack{90^\circ\text{ threaded}\\ \text{elbow}}}\frac{\bar{V}_2^2}{2g}+K_{\text{exit}}\frac{\bar{V}_{2'}^2}{2g}$$

$$H_{L,12''} = \left[f\left(\frac{L}{D}\right)+K_{\substack{\text{re-entrant}\\ \text{inlet}}}+K_{\substack{\text{1/2 open}\\ \text{gate valve}}}+2K_{\substack{90^\circ\text{ threaded}\\ \text{elbow}}}\right]\frac{\bar{V}_2^2}{2g}+K_{\text{exit}}\frac{\bar{V}_{2'}^2}{2g} \tag{6}$$

(Note that the kinetic energy in the stream is dissipated when going from point 2' to point 2''. Thus, the correct velocity to use in the velocity head term is the velocity at 2'.)

Using the same data as in the previous solution, except for the velocity at 2' and the elevation at 2',

$$p_A = p_B + \rho g\left(z_{2''} - z_1 + H_{L,12''}\right)$$

$$= \left(2.8\text{E}5 \text{ Pa}\right)+\left(1.0\text{E}3 \text{ kg/m}^3\right)\left(9.8\text{E}0 \text{ m/s}^2\right)\left(1.4\text{E}1 \text{ m} - 6.0\text{E}0 \text{ m} + 3.2\text{E}1 \text{ m}\right)$$

$\boxed{p_A = 6.8\text{E}5 \text{ Pa}}$ (Same answer as found previously!)