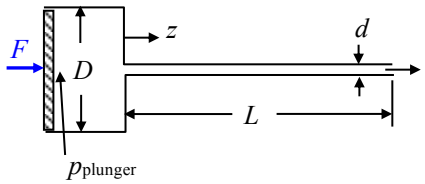
A hypodermic needle, with an inside diameter of 0.1 mm and a length of 25 mm is used to inject saline solution with a dynamic viscosity five times that of water. The plunger diameter is 10 mm and the maximum force that can be exerted by a thumb on the plunger is 45 N. Estimate the volume flow rate of saline that can be produced.

SOLUTION:



For a viscous, laminar, fully developed flow in a circular pipe (Poiseuille flow), the average velocity is

$$\bar{u} = \frac{d^2}{32\mu}\left(-\frac{dp}{dz}\right) \tag{1}$$

and the volumetric flow rate is:

$$Q = \bar{u}\frac{\pi d^2}{4} = \frac{\pi d^4}{128\mu}\left(-\frac{dp}{dz}\right) \tag{2}$$

The pressure gradient, assuming fully developed flow in the needle, is:

$$\frac{dp}{dz} = \frac{\Delta p}{L} = \frac{p_{atm} - p_{plunger}}{L} = \frac{-p_{plunger,gage}}{L} \tag{3}$$

where $p_{plunger,gage}$ is:

$$p_{plunger,gage} = \frac{F}{\left(\pi D^2 / 4\right)} \tag{4}$$

Using the given data:

d = 0.1e-3 m
D = 10e-3 m
L = 25e-3 m
F = 45 N
μ = 5e-3 N·m/s
$\Rightarrow$ $p_{plunger,gage}$ = 5.73e5 Pa
$\Rightarrow$ dp/dz = -2.29e7 Pa/m
$\Rightarrow$ $\bar{u}$ = 1.43 m/s
$\Rightarrow$ $\boxed{Q = 1.13\text{e-}8\ \text{m}^3/\text{s} = 11.3\ \text{mm}^3/\text{s}}$

Check the Reynolds number to verify that the laminar flow assumption is ok.

$$\text{Re} = \frac{\rho \bar{u} d}{\mu} \quad (\text{Use } \rho \approx 1000\ \text{kg/m}^3.) \tag{5}$$

$\Rightarrow$ Re = 28.8 < 2300 $\Rightarrow$ The laminar flow assumption is justified!