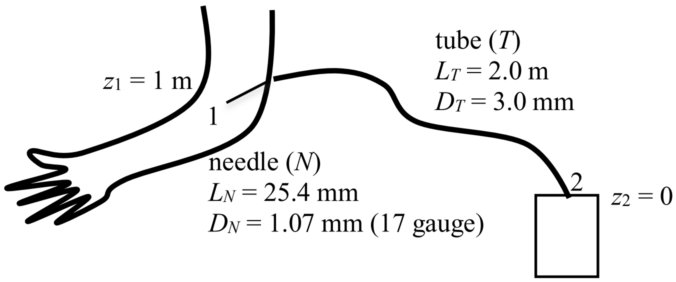
Consider the process of donating blood. Blood flows from a vein in which the pressure is greater than atmospheric, through a long small-diameter tube, and into a plastic bag that is essentially at atmospheric pressure. Based on fluid mechanics principles, estimate the amount of time it takes to donate a pint of blood. List all assumptions and show calculations.

SOLUTION:



First, a few assumptions:

1. Treat blood as a Newtonian fluid. Blood is actually slightly non-Newtonian with shear-thinning behavior, but we'll model it as Newtonian here for simplicity. Assume the density of blood is ρ = 1060 kg/m^3 and its dynamic viscosity is μ = 3.5 cP = 3.5*10^{-3} kg/(m.s). Hence, the kinematic viscosity is $\nu = \mu/\rho$ = 3.30*10^{-6} m^2/s.
2. Steady flow through the needle and tube. In fact, the real flow will be pulsatile due to the fluctuating pressure in the vein.
3. Constant pressure in the arm and collection bag. Again, the real flow will have periodic pressure variations in the arm.
4. The mean arterial pressure in the arm is at 93.3 mm Hg (gage). In practice, there are two values given for blood pressure: a systolic pressure and a diastolic pressure. The systolic pressure is the pressure when the heart is contracted while the diastolic pressure is when the heart is relaxed. A value of (systolic/diastolic) 120/80 mm Hg (gage) is within the normal range of blood pressures. The mean arterial pressure (MAP) is the average pressure over a cardiac cycle and can be approximated as: MAP = $p_{\text{diastolic}} + (1/3)*(p_{\text{systolic}} - p_{\text{diastolic}})$.
5. The pipe system consists of a needle, plastic tubing, and a plastic collection bag. A 17 gauge (1.07 mm inner diameter) needle diameter is often used for collecting blood. These needles are approximately 2.54 cm (1 in.) in length. The plastic tubing is assumed to be approximately 2.0 m in length with an inner diameter of 3.0 mm.
6. The flow in the needle and tube is laminar since the diameters are small.
7. The collection bag is located below the person's arm. We'll assume an elevation difference of 0.5 m.

Apply the Extended Bernoulli Equation from point 1 (in the vein) to point 2 (just upstream of the tube exit leading into the bag),

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_2=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_{L,1\to 2}+H_{S,1\to 2}\,, \tag{1}$$

where

p_1 = p_{arm} = 93.3 mm Hg (gage) (use the mean arterial pressure in the arm)
p_2 = 0 (gage) (discharging into a bag that is at atmospheric pressure)
$\bar{V}_1 \approx 0$ (blood speed in the vein is small compared to the speed in the needle and tube)
$\bar{V}_2 = \bar{V}_T$ (blood speed just before the exit of the tube)
α_2 = α_T = 2 (because the tube diameter is small, assume the flow is laminar at the tube exit)
z_1 = 0.5 m (assume the person's arm is 0.5 m above the bag)
z_2 = 0
$H_{S,1\to 2}$ = 0 (no fluid machinery in the process)

$$H_{L,1\to 2}=\frac{\bar{V}_N^2}{2g}\left[f_N\left(\frac{L_N}{D_N}\right)+K_{\text{entrance}}+K_{\text{expansion}}\right]+\frac{\bar{V}_T^2}{2g}\left[f_T\left(\frac{L_T}{D_T}\right)\right] \tag{2}$$

(major losses in the needle and tube and a minor loss at the inlet and at the transition from the need to the tube)

Note that,

$$\bar{V}_N = \frac{Q}{\frac{\pi}{4}D_N^2} = \frac{4Q}{\pi D_N^2} \quad \text{and} \quad \bar{V}_T = \frac{Q}{\frac{\pi}{4}D_T^2} = \frac{4Q}{\pi D_T^2} \tag{3}$$

$$f_N = \frac{64}{\mathrm{Re}_{D_N}} = \frac{64\nu}{\bar{V}_N D_N} = \frac{16\pi\nu D_N}{Q} \quad \text{and} \quad f_T = \frac{64}{\mathrm{Re}_{D_T}} = \frac{64\nu}{\bar{V}_T D_T} = \frac{16\pi\nu D_T}{Q} \quad \text{(since the flow is laminar)} \tag{4}$$

Substitute and simplify,

$$\alpha_T \frac{\bar{V}_T^2}{2g} = \left(\frac{p_{g,\mathrm{arm}}}{\rho g} + z_1\right) - \frac{\bar{V}_N^2}{2g}\left[f_N\left(\frac{L_N}{D_N}\right) + K_{\mathrm{entrance}} + K_{\mathrm{expansion}}\right] - \frac{\bar{V}_T^2}{2g}\left[f_T\left(\frac{L_T}{D_T}\right)\right], \tag{5}$$

$$\left(\frac{p_{g,\mathrm{arm}}}{\rho g} + z_1\right) = \frac{8Q^2}{\pi^2 g D_N^4}\left(\frac{16\pi\nu L_N}{Q} + K_{\mathrm{entrance}} + K_{\mathrm{expansion}}\right) + \frac{8Q^2}{\pi^2 g D_T^4}\left(\frac{16\pi\nu L_T}{Q} + \alpha_T\right), \tag{6}$$

$$\left(\frac{p_{g,\mathrm{arm}}}{\rho g} + z_1\right) = \frac{8Q^2}{\pi^2 g D_N^4}\frac{16\pi\nu L_N}{Q} + \frac{8Q^2}{\pi^2 g D_T^4}\frac{16\pi\nu L_T}{Q} + \frac{8Q^2}{\pi^2 g D_N^4}\left(K_{\mathrm{entrance}} + K_{\mathrm{expansion}}\right) + \frac{8Q^2}{\pi^2 g D_T^4}\alpha_T\,, \tag{7}$$

$$\left(\frac{p_{g,\mathrm{arm}}}{\rho g} + z_1\right) = \frac{128\nu Q}{\pi g}\left(\frac{L_N}{D_N^4} + \frac{L_T}{D_T^4}\right) + \frac{8Q^2}{\pi^2 g}\left(\frac{K_{\mathrm{entrance}} + K_{\mathrm{expansion}}}{D_N^4} + \frac{\alpha_T}{D_T^4}\right), \tag{8}$$

$$\underbrace{\frac{8}{\pi^2 g}\left(\frac{K_{\mathrm{entrance}} + K_{\mathrm{expansion}}}{D_N^4} + \frac{\alpha_T}{D_T^4}\right)}_{=A} Q^2 + \underbrace{\frac{128\nu}{\pi g}\left(\frac{L_N}{D_N^4} + \frac{L_T}{D_T^4}\right)}_{=B} Q + \underbrace{-\left(\frac{p_{g,\mathrm{arm}}}{\rho g} + z_1\right)}_{=C} = 0\,. \tag{9}$$

Using the given data,

g = 9.81 m/s^2
K_{entrance} = 0.78 (re-entrant inlet)
$K_{\mathrm{expansion}}$ = 0.8 (area ratio = $(D_N/D_T)^2$ = 0.126)
D_N = 1.07*10^{-3} m
α_T = 2 (laminar flow at tube exit)
D_T = 3.0*10^{-3} m
ν = 3.30*10^{-6} m^2/s
L_N = 2.54*10^{-2} m
L_T = 2.0 m
$p_{g,\mathrm{arm}}$ = $\rho g H$ = (13.6*1000 kg/m^3)(9.81 m/s^2)(93.3*10^{-3} m Hg) = 12.5*10^3 Pa
ρ = 1060 kg/m^3
z_1 = 0.5 m

$\Rightarrow$ A = 1.03*10^{11} s^2/m^5, B = 6.07*10^5 s/m^2, C = -1.70 m

$\Rightarrow$ Q = 2.07*10^{-6} m^3/s $\Rightarrow$ $\bar{V}_N$ = 2.31 m/s, $\bar{V}_T$ = 0.29 m/s $\Rightarrow$ Re_{DN} = 748, Re_{DT} = 266 $\Rightarrow$ The laminar flow assumptions are good ones!

One pint is equivalent to V_{collect} = 4.73*10^{-4} m^3, thus the expected time required to collect one pint of blood is,

$$T = \frac{V_{\mathrm{collect}}}{Q} \Rightarrow \boxed{T = 229 \text{ s} = 3.8 \text{ min.}}$$

In practice, the time required to donate a pint of blood is approximately 8 – 10 minutes, so this prediction, although in the right ballpark, is too small when compared to reality. Two assumptions likely factor into this error. First, we've assumed fully developed flow in the needle, which is most likely not the case. The pressure drop in the needle will be larger than what we've predicted from our fully developed flow model and thus the flow rate will decrease and the predicted donation time will increase. Secondly, there will be additional losses due to the bends in the tubing and, especially, due to the clamp located on the tubing to make it easier to stop the flow, if needed. These additional minor losses aren't negligible and will contribute to make the flow rate smaller.