A liquid with a specific gravity of 0.95 flows steadily at an average velocity of 10 m/s through a horizontal, smooth tube of diameter 5 cm. The fluid pressure is measured at 1 m intervals along the pipe as follows:

x [m]	0	1	2	3	4	5	6
p [kPa]	304	273	255	240	226	213	200

a. Estimate the average wall shear stress, in Pa, in the fully developed region of the pipe.
b. What is the approximate wall shear stress between stations 1 and 2? State any significant assumptions you make.

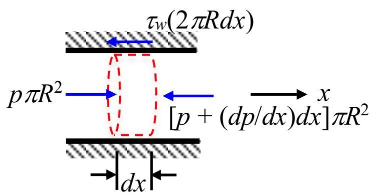
SOLUTION:

First determine the fully developed region by examining the pressure gradient in the pipe. The pressure gradient is constant in the fully developed region.

x [m]	0	1	2	3	4	5	6
p [kPa]	304	273	255	240	226	213	200
dp/dx [kPa/m]		-31	-18	-15	-14	-13	-13

Hence, the fully developed region starts at x = 4 m where the pressure drop remains constant at dp/dx = -13 kPa/m.

To determine the average wall shear stress in the pipe, apply the linear momentum equation in the x direction to the control volume shown in the figure below.



$$\frac{d}{dt}\int_{CV} u_x \rho \, dV + \int_{CS} u_x \left(\rho \mathbf{u}_{rel} \cdot d\mathbf{A}\right) = F_{S,x} + F_{B,x}\,, \tag{1}$$

where,

$$\frac{d}{dt}\int_{CV} u_x \rho \, dV = 0 \quad \text{(steady flow)}, \tag{2}$$

$$F_{B,x} = 0 \quad \text{(no body forces in } x\text{-direction)}, \tag{3}$$

$$F_{S,x} = p\pi R^2 - \left(p + \frac{dp}{dx}dx\right)\pi R^2 - \overline{\tau}_w\left(2\pi R dx\right) = -\frac{dp}{dx}dx\pi R^2 - \overline{\tau}_w\left(2\pi R dx\right), \tag{4}$$

$$\int_{CS} u_x \left(\rho \mathbf{u}_{rel} \cdot d\mathbf{A}\right) = 0 \tag{5}$$

(since for a fully-developed flow, the inlet and outlet velocity profiles are identical)

Substitute and simplify,

$$0 = -\frac{dp}{dx}dx\pi R^2 - \overline{\tau}_w\left(2\pi R dx\right), \tag{6}$$

$$\boxed{\overline{\tau}_w = -\frac{R}{2}\frac{dp}{dx}}. \tag{7}$$

Using the given data,

R = (0.05/2) m = 0.025 m

dp/dx = -13 kPa/m

$$\boxed{\Rightarrow \overline{\tau}_w = 163 \text{ Pa}}$$

For part (b), apply the same linear momentum equation, except that between stations 1 and 2, the velocity profile is not fully developed, hence the momentum flux term in the linear momentum equation (Eq. (5)) won't be zero. However, if the flow is turbulent, as would be expected for such a large velocity and assuming a liquid viscosity similar to that of water, the velocity profile will not change considerably as the flow continues downstream in the entrance region. The reason for this is that a turbulent velocity profile already looks like an average velocity profile due to the radial mixing associated with turbulence. Hence, although the momentum flux term isn't exactly zero, it is expected to be small in comparison to the pressure gradient term. As a result, even in the entrance region the average wall shear stress may be found using,

$$\boxed{\bar{\tau}_w \approx -\frac{R}{2}\frac{dp}{dx}}. \tag{8}$$

Using the given data between stations 1 and 2,

R = (0.05/2) m = 0.025 m

dp/dx = -18 kPa/m

$$\boxed{\Rightarrow \bar{\tau}_w = 225\ \text{Pa}}$$