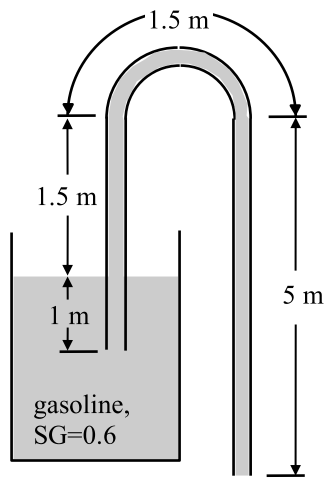
Gasoline at 20 °C is being siphoned from a tank through a rubber hose having an inside diameter of 25 mm. The roughness for the hose is 0.01 mm.

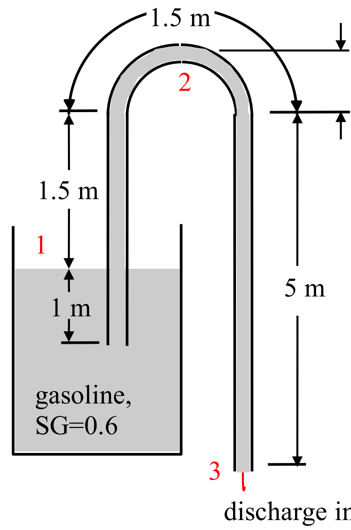
1. What is the volumetric flow rate of the gasoline through the hose?
2. What is the minimum pressure in the hose and where does it occur?

You may neglect minor losses. The kinematic viscosity of gasoline is 4.294e-7 m^2/s.



SOLUTION:

Apply the Extended Bernoulli Equation from points 1 to 3.



radius = circumference/(2π) = (2*1.5 m)/(2π) = 0.48 m

Assume the bend in the hose is circular and that the half circumference is 1.5 m.

$$\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_3=\left(\frac{p}{\rho g}+\alpha\frac{\bar{V}^2}{2g}+z\right)_1-H_L+H_S \tag{1}$$

where

$p_1 = p_3 = p_{atm}$ (2)

$\bar{V}_1 \approx 0$ (surface of a large tank) (3)

$\bar{V}_3 = ?$ ($\alpha_3 \approx 1$, assuming turbulent flow in the hose) (4)

$z_1 - z_3 = 3.5$ m (5)

$H_S = 0$ (6)

$H_L = f\left(\frac{L}{D}\right)\frac{\bar{V}_3^2}{2g}$ (Neglecting minor losses.) (7)

where f may be found from the Moody diagram. The relative roughness is

$$\frac{\varepsilon}{D}=\frac{0.01\text{ mm}}{25\text{ mm}}=4*10^{-4} \tag{8}$$

Assuming fully turbulent flow so that f is not a function of the Reynolds number, the Moody diagram gives, $f = 0.016$.

Substitute into Eq. (1) and solve for $\bar{V}_3$.

$$\frac{\bar{V}_3^2}{2g}\left[1+f\left(\frac{L}{D}\right)\right]=\left(z_1-z_3\right) \tag{9}$$

$$\bar{V}_3=\sqrt{\frac{2g\left(z_1-z_3\right)}{1+f\left(\frac{L}{D}\right)}} \tag{10}$$

Using the given data,

$g = 9.81\ \text{m/s}^2$

$z_1 - z_3 = 3.5\ \text{m}$

$f = 0.016$

$L = 1\ \text{m} + 1.5\ \text{m} + 1.5\ \text{m} + 5\ \text{m} = 9\ \text{m}$

$D = 0.025\ \text{m}$

$\Rightarrow\ \bar{V}_3 = 3.2\ \text{m/s}$

Check the Reynolds number to verify the fully turbulent assumption.

$$\text{Re}_D = \frac{\bar{V}_3 D}{\nu} = \frac{(3.2\ \text{m/s})(0.025\ \text{m})}{(4.294*10^{-7}\ \text{m}^2/\text{s})} = 1.9*10^5 \tag{11}$$

The given relative roughness and this Reynolds number puts the flow in the fully turbulent range so the assumptions made in the problem are consistent.

The flow rate is given by,

$$Q = \bar{V}_3 \frac{\pi D^2}{4} = (3.2\ \text{m/s})\frac{\pi(0.025\ \text{m})^2}{4} \Rightarrow \boxed{Q = 1.57*10^{-3}\ \text{m}^3/\text{s}} \tag{12}$$

The minimum pressure occurs near point 2 in the figure shown previously. Apply the Extended Bernoulli Equation from points 2 to 3.

$$\left(\frac{p}{\rho g} + \alpha\frac{\bar{V}^2}{2g} + z\right)_3 = \left(\frac{p}{\rho g} + \alpha\frac{\bar{V}^2}{2g} + z\right)_2 - H_L + H_S \tag{13}$$

where

$p_2 = ?$

$p_3 = p_{\text{atm}}$ (14)

$\bar{V}_2 = \bar{V}_3$ $(\alpha_2 \approx \alpha_3 \approx 1$ (15)

$z_2 - z_3 = 0.48\ \text{m} + 5\ \text{m} = 5.48\ \text{m}$ (16)

$H_S = 0$ (17)

$$H_L = f\left(\frac{L}{D}\right)\frac{\bar{V}_3^2}{2g} \tag{18}$$

where $f = 0.016$ was found previously.

Substitute into Eq. (13) and solve for p_2.

$$p_2 = p_3 + \rho g(z_3 - z_2) + f\left(\frac{L}{D}\right)\tfrac{1}{2}\rho\bar{V}_3^2 \tag{19}$$

Using the given data,

$p_3 = 101\ \text{kPa (abs)}$

$\rho = 0.6(1000\ \text{kg/m}^3) = 600\ \text{kg/m}^3$

$g = 9.81\ \text{m/s}^2$

$z_3 - z_2 = -5.48\ \text{m}$

$f = 0.016$

$L = (1.5\ \text{m})/2 + 5\ \text{m} = 5.75\ \text{m}$

$D = 0.025\ \text{m}$

$\bar{V}_3 = 3.2\ \text{m/s}$

$\Rightarrow\ \boxed{p_2 = 80.0\ \text{kPa (abs)}}$