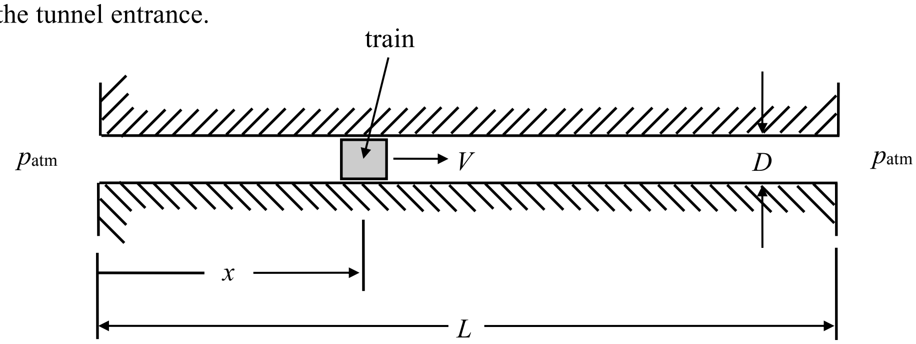
A train travels through a tunnel as shown in the figure. The train and tunnel are both circular in cross section. The tunnel has a diameter of D=3 m, a total length of L=2000 m, and walls comprised of concrete. The clearance between the train and the tunnel wall is small so that it may be assumed that the air in front of the train is pushed through the tunnel with the same speed as the train, V=20 m/s.

1. Determine the pressure difference between the front and rear of the train when the train is a distance, x, from the tunnel entrance.
2. Determine the power, P, required to produce the air flow in the tunnel when the train is a distance, x, from the tunnel entrance.



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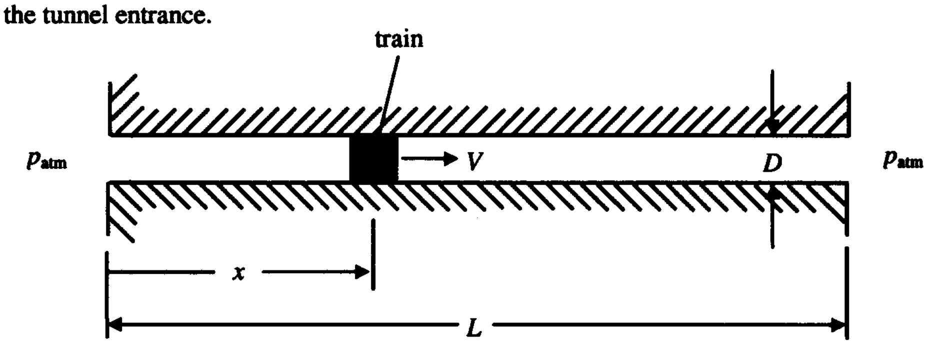
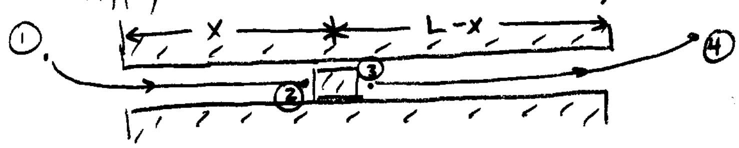


SOLUTION:

· Apply the Extended Bernoulli Eqn from ① to ② and from ③ to ④:



$$\left(\frac{p}{\rho g} + \alpha\frac{\bar{V}^2}{2g} + z\right)_2 = \left(\frac{p}{\rho g} + \alpha\frac{\bar{V}^2}{2g} + z\right)_1 - H_{L_{1\to2}} + H_{S_{1\to2}}$$

$$\left(\frac{p}{\rho g} + \alpha\frac{\bar{V}^2}{2g} + z\right)_4 = \left(\frac{p}{\rho g} + \alpha\frac{\bar{V}^2}{2g} + z\right)_3 - H_{L_{3\to4}} + H_{S_{3\to4}}$$

where
$p_1 = p_{atm}$ $\quad p_3 = ?$
$p_2 = ?$ $\quad p_4 = p_{atm}$

$\bar{V}_1 = 0$ $\quad \bar{V}_3 = V$
$\bar{V}_2 = V$ $\quad \bar{V}_4 = 0$

since flow is expected to be turbulent, $\alpha_2 = \alpha_3 \approx 1$

$$Re = \frac{VD}{\nu_{air}} = \frac{(20\,m/s)(3\,m)}{(1.5\times10^{-5}\,m^2/s)} = 4\times10^6$$

$z_1 = z_2 = z_3 = z_4$

$H_{L1\text{-}2} = f\left(\frac{x}{D}\right)\frac{V^2}{2g}$ $\quad H_{S_{1\text{-}2}} = 0$

$H_{L3\text{-}4} = f\left(\frac{L-x}{D}\right)\frac{V^2}{2g}$ $\quad H_{S_{3\text{-}4}} = 0$

NOTE: Neglecting entrance & exit losses since tunnel is so long.

SOLUTION...

$$\frac{p_2}{\rho g} + \frac{V^2}{2g} = \frac{p_{atm}}{\rho g} - f\left(\frac{x}{D}\right)\frac{V^2}{2g}$$

$$\frac{p_{atm}}{\rho g} = \frac{p_3}{\rho g} + \frac{V^2}{2g} - f\left(\frac{L-x}{D}\right)\frac{V^2}{2g}$$

$$\Rightarrow \quad \frac{p_2 - p_{atm}}{\rho g} = -\left(1 + f\left(\frac{x}{D}\right)\right)\frac{V^2}{2g}$$

$$\frac{p_3 - p_{atm}}{\rho g} = \left(f\left(\frac{L-x}{D}\right) - 1\right)\frac{V^2}{2g}$$

$$\Rightarrow \quad \frac{p_3 - p_2}{\rho g} = \left[f\left(\frac{L-x}{D}\right) - 1 + 1 + f\left(\frac{x}{D}\right)\right]\frac{V^2}{2g}$$

$$= f\left(\frac{L}{D}\right)\frac{V^2}{2g}$$

$$\boxed{\therefore \quad p_3 - p_2 = \frac{1}{2}\rho f\left(\frac{L}{D}\right)V^2}$$

- For $\rho = 1.2\ \mathrm{kg/m^3}$
  - $f = 0.016$ found from Moody chart w/
    - $\epsilon = 1.7\times10^{-3}\,\mathrm{m}$ (concrete)
    - $Re = 4\times10^6$ (from previous page)
    - $D = 3\,\mathrm{m}$
    - $\Rightarrow \epsilon/D = 5.7\times10^{-4}$
  - $L = 2000\,\mathrm{m}$
  - $D = 3\,\mathrm{m}$
  - $V = 20\,\mathrm{m/s}$

$$\boxed{\Rightarrow p_2 - p_1 = 2560\,\mathrm{Pa}}$$

- Power is given by

$$P = F \cdot V = (\Delta p)\left(\frac{\pi D^2}{4}\right)V$$

For $\left.\begin{array}{l}\Delta p = 2560\,\mathrm{Pa}\\ D = 3\,\mathrm{m}\\ V = 20\,\mathrm{m/s}\end{array}\right\} \Rightarrow \boxed{P = 360\,\mathrm{KW}}$