The Mach number and temperature upstream of a shock wave are 2 and 7 °C, respectively. What is the air speed, relative to the shock wave, downstream of the shock wave?

SOLUTION:

Use the normal shock relations to determine the downstream Mach number.

$$\mathrm{Ma}_2^2 = \frac{(k-1)\mathrm{Ma}_1^2 + 2}{2k\mathrm{Ma}_1^2 - (k-1)} \Rightarrow \underline{\mathrm{Ma}_2 = 0.58} \qquad (1)$$

where k = 1.4 and Ma_1 = 2.

Determine the stagnation temperature upstream of the shock wave.

$$\frac{T_1}{T_{01}} = \left(1 + \frac{k-1}{2}\mathrm{Ma}_1^2\right)^{-1} \Rightarrow \underline{T_{01} = 504\ \mathrm{K}} \qquad (2)$$

where T_1 = (273 + 7) K = 280 K.

Note that the stagnation temperature remains constant across a shock wave, so $\underline{T_{02} = T_{01}}$. Use the downstream stagnation temperature and downstream Mach number to determine the downstream static temperature:

$$\frac{T_2}{T_{02}} = \left(1 + \frac{k-1}{2}\mathrm{Ma}_2^2\right)^{-1} \Rightarrow \underline{T_2 = 473\ \mathrm{K}} \qquad (3)$$

Use the definition of the Mach number and the speed of sound for an ideal gas to determine the air speed downstream of the shock wave:

$$V_2 = \mathrm{Ma}_2 c_2 \Rightarrow V_2 = \mathrm{Ma}_2\sqrt{kRT_2} \Rightarrow \boxed{V_2 = 252\ \mathrm{m/s}} \qquad (4)$$

where R_{air} = 287 J/(kg.K).