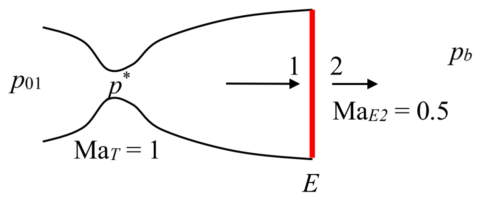
For the purposes of an experiment, we wish to design a de Laval nozzle which will be supplied from a compressed air reservoir (specific heat ratio of 1.4). It is required that:

1. there is a normal shock across the exit of the diffuser, and
2. the jet emerging downstream of the shock should have a Mach number of 0.5.

Find:

a. the ratio of the cross-sectional area at the diffuser exit to the cross-sectional area of the throat,
b. the ratio of the ambient pressure downstream of the shock to the pressure in the compressed air reservoir, and
c. the ratio of the ambient pressure downstream of the shock to the throat pressure.

SOLUTION:



The Mach number just upstream of the shock wave at the exit may be found using the normal shock relations,

$$\mathrm{Ma}_{E2}^2 = \frac{(k-1)\mathrm{Ma}_{E1}^2 + 2}{2k\mathrm{Ma}_{E1}^2 - (k-1)} \Rightarrow \underline{\mathrm{Ma}_{E1} = 2.6457} \quad (1)$$

The ratio of the cross-sectional area at the diffuser exit to the cross-sectional area of the throat may be found using the isentropic sonic area ratio and the Mach number just upstream of the shock,

$$\frac{A_E}{A_T} = \frac{A_E}{A^*} = \frac{1}{\mathrm{Ma}_{E1}}\left(\frac{1+\frac{k-1}{2}\mathrm{Ma}_{E1}^2}{1+\frac{k-1}{2}}\right)^{\frac{k+1}{2(k-1)}} \Rightarrow \boxed{A_E/A_T = 3.0236} \quad (2)$$

Note that since the flow at the exit is supersonic, the throat must be at a sonic Mach number.

The pressure ratio, p_b/p_{01}, is given by,

$$\frac{p_b}{p_{01}} = \left(\frac{p_b}{p_{E2}}\right)\left(\frac{p_{E2}}{p_{E1}}\right)\left(\frac{p_{E1}}{p_{01}}\right) \Rightarrow \boxed{p_b/p_{01} = 0.3736} \quad (3)$$

where

$$\frac{p_b}{p_{E2}} = 1 \ \text{(since } \mathrm{Ma}_{E2} < 1\text{)} \quad (4)$$

$$\frac{p_{E2}}{p_{E1}} = \frac{2k}{k+1}\mathrm{Ma}_{E1}^2 - \frac{k-1}{k+1} \Rightarrow p_{E2}/p_{E1} = 7.9997 \ \text{(normal shock relations)} \quad (5)$$

$$\frac{p_{E1}}{p_{01}} = \left(1+\frac{k-1}{2}\mathrm{Ma}_{E1}^2\right)^{\frac{k}{1-k}} \Rightarrow p_{E1}/p_{01} = 0.0467 \ \text{(isentropic stagnation pressure ratio)} \quad (6)$$

The pressure ratio, p_b/p^*, is given by,

$$\frac{p_b}{p^*} = \left(\frac{p_b}{p_{01}}\right)\left(\frac{p_{01}}{p^*}\right) \Rightarrow \boxed{p_b/p^* = 0.7071} \quad (7)$$

where

$$\frac{p^*}{p_{01}} = \left(1+\frac{k-1}{2}\right)^{\frac{k}{1-k}} \Rightarrow p^*/p_{01} = 0.5283 \ \text{(isentropic stagnation pressure ratio)} \quad (8)$$