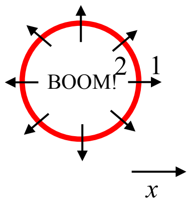
According to a newspaper article, at the center of a 12,600 lb_m “Daisy-Cutter” bomb explosion the overpressure in the air is approximately 1000 psi. Estimate:

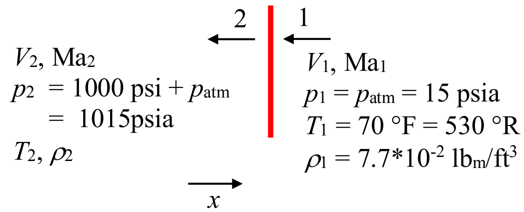
a. the speed of the resulting shock wave into the surrounding air,
b. the wind speed following the shock wave,
c. the temperature after the shock wave has passed, and
d. the air density after the shock wave has passed.

SOLUTION:

Change the frame of reference from one that is fixed to the ground to one that is fixed to the wave as shown in the schematic below. Treat the explosion shock wave as a normal shock.



→ Change the frame of reference so the wave appears stationary (substract the velocity of the wave to all velocities.) →



The pressure ratio across the wave is:

$$\frac{p_2}{p_1} = \frac{1015 \text{ psia}}{15 \text{ psia}} = 69$$

Using the normal shock relations:

$$\begin{aligned} p_2/p_1 = 69 \;\Rightarrow\; & \mathrm{Ma}_1 = 7.7 \\ \Rightarrow\; & \mathrm{Ma}_2 = 0.4 \\ \Rightarrow\; & T_2/T_1 = 12.5 \\ \Rightarrow\; & \rho_2/\rho_1 = 5.5 \end{aligned}$$

Now determine the unknown quantities.

$$V_1 = \mathrm{Ma}_1\sqrt{\gamma R T_1} = (7.7)\sqrt{(1.4)\left(53.3 \tfrac{\text{ft}\cdot\text{lb}_f}{\text{lb}_m\cdot\ \text{R}}\right)(530\ \ \text{R})}$$

$$\therefore V_1 = 8700 \text{ ft/s} \quad \text{(Note that is the velocity w/r/t the wave.)}$$

$$T_2 = \left(\frac{T_2}{T_1}\right)T_1 = (12.5)(530\ \ \text{R})$$

$$\boxed{\therefore T_2 = 6600\ \ \text{R} = 6100\ \ \text{F}}$$

$$\rho_2 = \left(\frac{\rho_2}{\rho_1}\right)\rho_1 = (5.5)\left(7.7*10^{-2}\ \text{lb}_m/\text{ft}^3\right)$$

$$\boxed{\therefore \rho_2 = 0.42\ \text{lb}_m/\text{ft}^3}$$

$$V_2 = \mathrm{Ma}_2\sqrt{\gamma R T_2} = (0.4)\sqrt{(1.4)\left(53.3 \tfrac{\text{ft}\cdot\text{lb}_f}{\text{lb}_m\cdot\ \text{R}}\right)(6600\ \ \text{R})}$$

$$\therefore V_2 = 1600 \text{ ft/s} \quad \text{(Note that is the velocity w/r/t the wave.)}$$

To determine the shock and downstream wind speed with respect to the ground, we must change back to our original frame of reference.

$$V_{\substack{\text{shock,}\\\text{w/r/t ground}}} = V_1$$

$$\boxed{\therefore V_{\substack{\text{shock,}\\\text{w/r/t ground}}} = 8700\text{ ft/s}}$$

$$V_{\substack{\text{downstream wind,}\\\text{w/r/t ground}}} = \underbrace{V_{\substack{\text{downstream wind,}\\\text{w/r/t shock}}}}_{=-V_2} + V_{\substack{\text{shock,}\\\text{w/r/t ground}}}$$

$$\boxed{\therefore V_{\substack{\text{downstream wind,}\\\text{w/r/t ground}}} = 7100\text{ ft/s}}$$