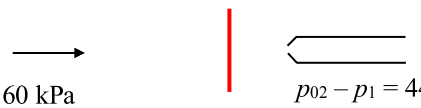
A stagnation tube is placed in a supersonic flow in which the static pressure and temperature far upstream are 60 kPa (abs) and -20 °C. The difference between the stagnation pressure measured by the stagnation tube and the upstream static pressure is 449 kPa. Determine the upstream Mach number and velocity of the flow.

SOLUTION:

Since there is no throat upstream of the stagnation tube, there must be a shock wave that forms in order to slow the flow from supersonic to subsonic conditions, and eventually stagnation conditions at the inlet to the stagnation tube.



p_1 = 60 kPa
T_1 = -20 °C = 253 K
γ = 1.4, R = 287 J/(kg·K)

$p_{02} - p_1$ = 449 kPa

Re-arrange the given conditions in order to solve for the upstream Mach number.

$$p_{02} - p_1 = \left(\frac{p_{02}}{p_1} - 1\right) p_1 = \left(\frac{p_{02}}{p_2}\frac{p_2}{p_1} - 1\right) p_1 = 449 \text{ kPa} \tag{1}$$

where

$$\text{Ma}_2^2 = \frac{(\gamma - 1)\text{Ma}_1^2 + 2}{2\gamma \text{Ma}_1^2 - (\gamma - 1)} \tag{2}$$

$$\frac{p_2}{p_{02}} = \left(1 + \frac{\gamma - 1}{2}\text{Ma}_2^2\right)^{\frac{\gamma}{1-\gamma}} \tag{3}$$

$$\frac{p_2}{p_1} = \frac{2\gamma}{\gamma + 1}\text{Ma}_1^2 - \frac{\gamma - 1}{\gamma + 1} \tag{4}$$

Iterate to a converged solution using the following approach.

1. Assume a value for Ma_1.
2. Determine Ma_2 using Eq. (2).
3. Determine p_2/p_{02} using Eq. (3).
4. Determine p_2/p_1 using Eq. (4).
5. Substitute the values calculated in the previous steps into the left-hand side of Eq. (1), along with p_1 = 60 kPa.
6. Check to see if the calculation from step 5 equals the right-hand side of Eq. (1). If the calculation is smaller than the right-hand side of Eq. (1) then the assumed Ma_1 was too small and a larger Ma_1 should be chosen. If the calculation is larger than the right-hand side of Eq. (1) then the assumed Ma_1 was too large and a smaller Ma_1 should be chosen. Steps 2 through 6 should be repeated until a converged solution results.

Following the previous iterative procedure:

$$\boxed{\text{Ma}_1 = 2.493}$$

and

$$\boxed{V_1 = \text{Ma}_1\sqrt{\gamma R T_1} = 795 \text{ m/s}} \tag{5}$$