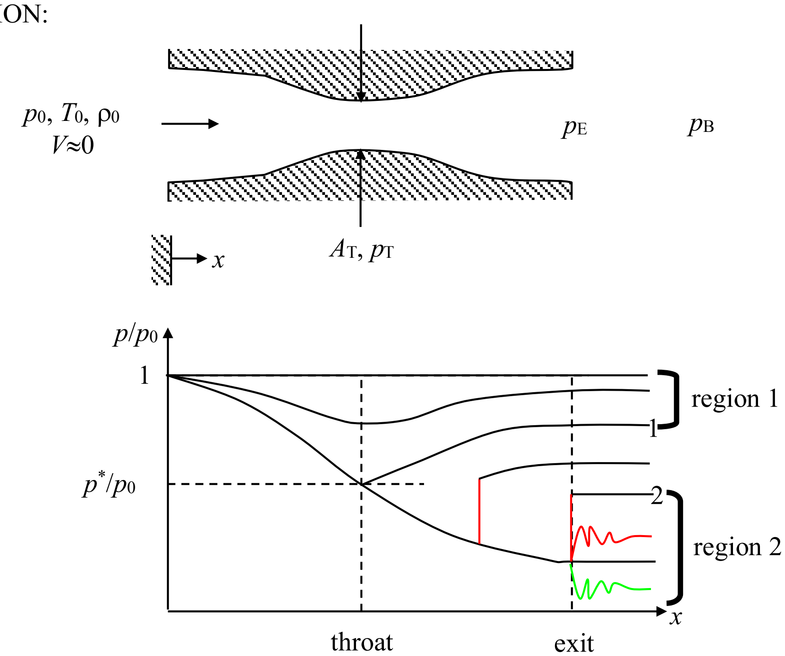
A converging-diverging nozzle, with $A_e/A_t = 1.633$, is designed to operate with atmospheric pressure at the exit plane. Determine the range(s) of stagnation pressures for which the nozzle will be free from normal shocks.

SOLUTION:



There will be two ranges of back pressures that will not produce shock waves *within* the C-D nozzle. In region 1 shown above, the entire flow remains subsonic (with possible sonic flow at the throat). In region 2 the flow is subsonic in the converging section, sonic at the throat, then subsonic throughout the diverging section. Shock waves and expansion fans may occur *outside* of the C-D nozzle in region 2.

Consider pressure curve 1 indicated in the figure above. For this case the exit Mach number is given by:

$$\frac{A_E}{A_T} = \frac{A_E}{A^*} = 1.633 = \frac{1}{\text{Ma}_E}\left(\frac{1+\frac{\gamma-1}{2}\text{Ma}_E^2}{1+\frac{\gamma-1}{2}}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \tag{1}$$

Solve for the subsonic exit Mach number to get:

$\text{Ma}_E = 0.387$

Now use the isentropic stagnation pressure ratio to determine the reservoir stagnation pressure for these conditions.

$$\frac{p_E}{p_0} = \left(1+\frac{\gamma-1}{2}\text{Ma}_E^2\right)^{\frac{\gamma}{1-\gamma}} \Rightarrow p_0 = 1.11 p_E = 112 \text{ kPa} \ \ (\text{where } p_E = p_{atm} = 101 \text{ kPa}) \tag{2}$$

Hence, the nozzle will be shock free for:

$\boxed{p_{atm} \le p_0 \le 1.11 p_{atm} \text{ or } 101 \text{ kPa} \le p_0 \le 112 \text{ kPa}}$

Now consider pressure curve 2 indicated in the figure above. For this case a normal shock wave occurs at the nozzle exit plane. Just upstream of the shock wave the Mach number can be found using the sonic area ratio.

$$\frac{A_{E1}}{A^*} = \frac{A_E}{A_T} = 1.633 \quad \Rightarrow \quad \text{Ma}_{E1} = 1.96 \text{ (using the isentropic flow relations)}$$

$\Rightarrow$ $p_{E2}/p_{E1} = 4.3152$ (using the normal shock relations with $\text{Ma}_{E1} = 1.96$)
$\Rightarrow$ $p_{E1}/p_{01} = 0.1359$ (using the isentropic flow relations with $\text{Ma}_{E1} = 1.96$)

Now solve for p_{E2}/p_{01}.

$$\frac{p_{E2}}{p_{01}} = \left(\frac{p_{E2}}{p_{E1}}\right)\left(\frac{p_{E1}}{p_{01}}\right) = (4.3152)(0.1359) = 0.5864$$

Note that $p_{01} = p_0$ (the reservoir pressure) and $p_{E2} = p_{\text{atm}}$ (since the flow downstream of the shock is subsonic).

$\Rightarrow$ $p_0 = 1.7052\, p_{\text{atm}}$

Thus, normal shocks will not form in the C-D nozzle when:

$\boxed{p_0 > 1.71\, p_{\text{atm}} \text{ or } p_0 > 172 \text{ kPa}}$

To summarize, the C-D nozzle will remain shock-free for the following range of stagnation pressures:

$\boxed{p_{\text{atm}} \le p_0 \le 1.11 p_{\text{atm}} \text{ and } p_0 > 1.71\, p_{\text{atm}}}$