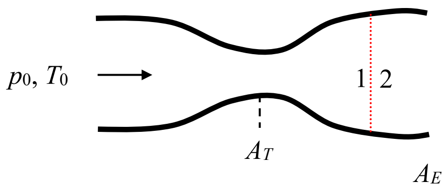
Air flows through a frictionless, adiabatic converging-diverging nozzle. The air in the reservoir feeding the nozzle has a pressure and temperature of 700 kPa (abs) and 500 K, respectively. The ratio of the nozzle exit to throat area is 11.91. A normal shock wave stands where the upstream Mach number is 3.0. Calculate the Mach number, the static temperature, and static pressure at the nozzle exit plane.

SOLUTION:



p_0 = 700 kPa (abs)
T_0 = 500 K
A_E/A_T = 11.91
Ma_1 = 3.0

Using the normal shock relations:

$$Ma_1 = 3.0 \Rightarrow \quad Ma_2 = 0.4752 \tag{1}$$
$$T_{02}/T_{01} = 1 \tag{2}$$
$$p_{02}/p_{01} = 0.3283 \tag{3}$$

The flow is isentropic from the reservoir to just upstream of the shock (location 1) so that:

$$p_{01} = p_0 \tag{4}$$
$$T_{01} = T_0 \tag{5}$$
$$A_1/A_1^* = 4.2346 \text{ (using } Ma_1 = 3.0) \tag{6}$$

The flow is also isentropic from just downstream of the shock (location 2) to the exit so that:

$$p_{0E} = p_{02} \tag{7}$$
$$T_{0E} = T_{02} \tag{8}$$
$$A_2/A_2^* = 1.390 \text{ (using } Ma_2 = 0.4752) \tag{9}$$

Combine the previous equations to get the exit stagnation conditions.

$$p_{0E} = p_{02} = \left(\frac{p_{02}}{p_{01}}\right)p_{01} = \left(\frac{p_{02}}{p_{01}}\right)p_0 = (0.3283)(700 \text{ kPa}) = 229.8 \text{ kPa} \tag{10}$$

$$T_{0E} = T_{02} = \left(\frac{T_{02}}{T_{01}}\right)T_{01} = \left(\frac{T_{02}}{T_{01}}\right)T_0 = (1)(500 \text{ K}) = 500 \text{ K} \tag{11}$$

Now determine the exit sonic area ratio (A_E/A^*) so that it can be used to determine the exit Mach number.

$$\frac{A_E}{A_2^*} = \left(\frac{A_E}{A_T}\right)\left(\frac{A_1^*}{A_1}\right)\left(\frac{A_2}{A_2^*}\right) = (11.91)\left(\frac{1}{4.2346}\right)(1.390) = 3.9094 \quad \text{(Note that } A_T = A_1^*\text{.)} \tag{12}$$

Use this area ratio and the isentropic flow sonic area relation to determine the exit Mach number. Note that the exit Mach number will be subsonic since the flow downstream of the shock wave is subsonic.

$$\frac{A_E}{A_2^*} = 3.9094 \Rightarrow \boxed{Ma_E = 0.15} \tag{13}$$

Use the isentropic flow relations with the exit Mach number to determine the stagnation temperature and pressure ratios.

$$\frac{T_E}{T_{0E}} = 0.9955 \quad \text{and} \quad \frac{p_E}{p_{0E}} = 0.9844 \tag{14}$$

Combine Eqns. (14) with Eqs. (10) and (11) to determine the exit static temperature and pressure.

$\boxed{T_E = 498 \text{ K}}$ and $\boxed{p_E = 226 \text{ kPa (abs)}}$