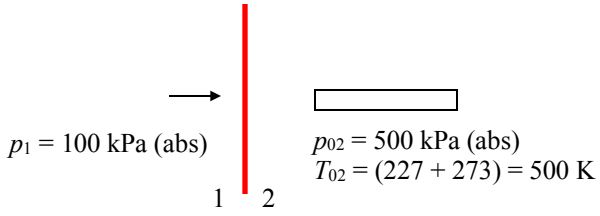
A total pressure probe is inserted into a supersonic air flow. A shock wave forms just upstream of the impact hole. The probe measures a total pressure of 500 kPa (abs) and the stagnation temperature at the probe head is 227 °C. The static pressure upstream of the shock is measured with a wall tap to be 100 kPa (abs).

a. Determine the Mach number of the incoming flow.
b. Determine the velocity of the incoming flow.
c. Sketch the process on a *T*-*s* diagram.

SOLUTION:



Determine the upstream Mach number by combining the isentropic pressure ratio and the stagnation pressure ratio across a normal shock.

$$\frac{p_1}{p_{01}} = \left(1+\frac{\gamma-1}{2}\mathrm{Ma}_1^2\right)^{\frac{\gamma}{1-\gamma}} \tag{1}$$

$$\frac{p_{02}}{p_{01}} = \left[\frac{(\gamma+1)\mathrm{Ma}_1^2}{2+(\gamma-1)\mathrm{Ma}_1^2}\right]^{\frac{\gamma}{\gamma-1}}\left[\frac{\gamma+1}{2\gamma\mathrm{Ma}_1^2-(\gamma-1)}\right]^{\frac{1}{\gamma-1}} \tag{2}$$

$$\frac{p_1}{p_{02}} = \left(\frac{p_1}{p_{01}}\right)\left(\frac{p_{01}}{p_{02}}\right) = \left(1+\frac{\gamma-1}{2}\mathrm{Ma}_1^2\right)^{\frac{\gamma}{1-\gamma}}\left[\frac{(\gamma+1)\mathrm{Ma}_1^2}{2+(\gamma-1)\mathrm{Ma}_1^2}\right]^{\frac{\gamma}{1-\gamma}}\left[\frac{\gamma+1}{2\gamma\mathrm{Ma}_1^2-(\gamma-1)}\right]^{\frac{1}{1-\gamma}} \tag{3}$$

Solve Eqn. (3) numerically for Ma_1 given that p_1 = 100 kPa and p_{02} = 500 kPa (and γ = 1.4).

$$\boxed{\mathrm{Ma}_1 = 1.87} \tag{4}$$

The velocity may be found from the Mach number and speed of sound on the upstream side of the shock wave.

$$V_1 = c_1\mathrm{Ma}_1 = \sqrt{\gamma RT_1}\,\mathrm{Ma}_1 \;\Rightarrow\; \boxed{V_1 = 643.1 \text{ m/s}} \tag{5}$$

where the upstream static temperature is found from the adiabatic stagnation temperature ratio and noting that $T_{01} = T_{02}$.

$$\frac{T_1}{T_{01}} = \left(1+\frac{\gamma-1}{2}\mathrm{Ma}_1^2\right)^{-1} \;\Rightarrow\; T_1 = 294.1 \text{ K} \tag{6}$$