

Air approaches a normal shock with  $T_1 = 18\text{ }^\circ\text{C}$ ,  $p_1 = 101\text{ kPa (abs)}$ , and  $V_1 = 766\text{ m/s}$ . The temperature immediately downstream from the shock is  $T_2 = 551\text{ K}$ .

1. Determine the velocity immediately downstream from the shock.
2. Determine the pressure change across the shock.
3. Calculate the corresponding pressure change for a frictionless, shockless deceleration between the same speeds and temperatures.

SOLUTION:

$$\begin{array}{ccc}
 p_1 = 101 \text{ kPa (abs)} & & \\
 T_1 = (18+273) \text{ K} & \longrightarrow & T_2 = 551 \text{ K} \\
 V_1 = 766 \text{ m/s} & & \\
 & \text{1} \quad | \quad \text{2} & 
 \end{array}$$

The velocity downstream of the shock may be found from conservation of energy.

$$c_p T_1 + \frac{1}{2} V_1^2 = c_p T_2 + \frac{1}{2} V_2^2 \quad (1)$$

$$V_2 = \sqrt{V_1^2 + 2c_p(T_1 - T_2)} \Rightarrow \boxed{V_2 = 254.3 \text{ m/s}} \quad (2)$$

using  $c_p = 1004 \text{ J/(kg}\cdot\text{K)}$ .

The pressure change across the shock may be found using the normal shock relations.

$$\Delta p = p_2 - p_1 = p_1 \left( \frac{p_2}{p_1} - 1 \right) \Rightarrow \boxed{\Delta p = 4.73 \cdot 10^5 \text{ Pa}} \quad (3)$$

where

$$\frac{p_2}{p_1} = \frac{2\gamma}{\gamma+1} \text{Ma}_1^2 - \frac{\gamma-1}{\gamma+1} \Rightarrow p_2/p_1 = 5.6880 \quad (4)$$

and

$$\text{Ma}_1 = \frac{V_1}{c_1} = \frac{V_1}{\sqrt{\gamma R T_1}} \Rightarrow \text{Ma}_1 = 2.24 \quad (5)$$

In addition,

$$\text{Ma}_2^2 = \frac{(\gamma-1)\text{Ma}_1^2 + 2}{2\gamma\text{Ma}_1^2 - (\gamma-1)} \Rightarrow \text{Ma}_2 = 0.54 \quad (6)$$

Note that we could have also simply used:

$$\text{Ma}_2 = \frac{V_2}{c_2} = \frac{V_2}{\sqrt{\gamma R T_2}} \Rightarrow \text{Ma}_2 = 0.54 \text{ (Same result as the previous one!)} \quad (7)$$

The corresponding pressure change for an isentropic deceleration between the same speeds may be found by combining isentropic stagnation pressure ratios,

$$\Delta p = p_2 - p_1 = p_1 \left( \frac{p_2}{p_1} - 1 \right) \Rightarrow \boxed{\Delta p_{\text{isentropic}} = 8.41 \cdot 10^5 \text{ Pa}} \quad (8)$$

where,

$$\frac{p_2}{p_1} = \frac{\left( \frac{p_2}{p_0} \right)}{\left( \frac{p_1}{p_0} \right)} = \frac{\left( 1 + \frac{\gamma-1}{2} \text{Ma}_2^2 \right)^{\frac{\gamma}{\gamma-1}}}{\left( 1 + \frac{\gamma-1}{2} \text{Ma}_1^2 \right)^{\frac{\gamma}{\gamma-1}}} \Rightarrow p_2/p_1 = 9.3253 \quad (9)$$