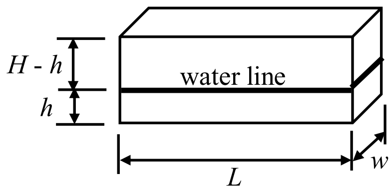
A barge weighing 8820 kN that is 10 m wide, 30 m long, and 7 m tall has come free from its tug boat in the Mississippi River. It is in a section of river that has a current of 1 m/s. In addition, there is a wind blowing straight upriver at 10 m/s . Assume that the drag coefficient is 1.3 for both the part of the barge in the wind as well as the part below the water. The drag coefficients for the water-exposed and air-exposed portions of the barge are based on the water and air wetted areas, respectively. Determine the speed at which the barge will be steadily moving. Is it moving upriver or downriver?

SOLUTION:



First determine the wetted areas above and below the waterline. Balancing forces in the vertical direction on the barge,

$$\sum F_{\text{vert}} = 0 = -W + \rho_{H_2O} g L w h, \tag{1}$$

where W is the barge weight and the second term on the right hand side is the buoyant force, with h being the draft of the barge (the depth below the water). Solving for h gives,

$$h = \frac{W}{\rho_{H_2O} g L w}. \tag{2}$$

Using the given values,

W = 8820 kN,
ρ_{H2O} = 1000 kg/m^3,
g = 9.81 m/s^2,
L = 30 m,
w = 10 m,
=> h = 3.00 m.

Now determine the wetted areas below and above water,

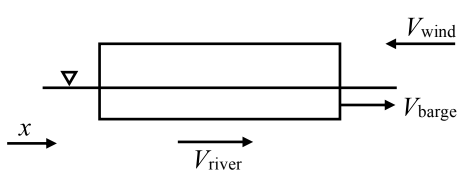
$$A_{\substack{\text{wetted,}\\ \text{below}}} = Lw + 2Lh + 2wh, \tag{3}$$

$$A_{\substack{\text{wetted,}\\ \text{above}}} = Lw + 2L(H-h) + 2w(H-h). \tag{4}$$

Using the given values,

$A_{\text{wetted,below}}$ = 540 m^2,
$A_{\text{wetted,above}}$ = 620 m^2.

Now balance forces in the horizontal direction. These forces include the drag caused by the river and the drag caused by the wind. Assume that the barge is moving in the same direction as the river (downstream), as shown in the figure below.



$$\sum F_x = 0 = c_{D,\text{below}} \tfrac{1}{2} \rho_{H_2O} \left(V_{\text{river}} - V_{\text{barge}}\right)^2 A_{\substack{\text{wetted,}\\ \text{below}}} - c_{D,\text{above}} \tfrac{1}{2} \rho_{\text{air}} \left(V_{\text{wind}} + V_{\text{barge}}\right)^2 A_{\substack{\text{wetted,}\\ \text{above}}}, \tag{5}$$

Solve for V_{barge}, noting that the drag coefficients are the same above and below the waterline (given in the problem statement),

$$\rho_{H_2O} \left(V_{\text{river}} - V_{\text{barge}}\right)^2 A_{\substack{\text{wetted,}\\ \text{below}}} = \rho_{\text{air}} \left(V_{\text{wind}} + V_{\text{barge}}\right)^2 A_{\substack{\text{wetted,}\\ \text{above}}}, \tag{6}$$

$$V_{\text{river}}^2 - 2V_{\text{river}}V_{\text{barge}} + V_{\text{barge}}^2 = \underbrace{\left(\frac{\rho_{\text{air}}}{\rho_{H_2O}}\right)\left(\frac{A_{\substack{\text{wetted,}\\\text{above}}}}{A_{\substack{\text{wetted,}\\\text{below}}}}\right)}_{=c}\left(V_{\text{wind}}^2 + 2V_{\text{wind}}V_{\text{barge}} + V_{\text{barge}}^2\right), \tag{7}$$

$$(1-c)V_{\text{barge}}^2 - 2\left(V_{\text{river}} + cV_{\text{wind}}\right)V_{\text{barge}} + V_{\text{river}}^2 - cV_{\text{wind}}^2 = 0, \tag{8}$$

$$\underbrace{1}_{=A}V_{\text{barge}}^2 + \underbrace{\frac{-2\left(V_{\text{river}} + cV_{\text{wind}}\right)}{(1-c)}}_{=B}V_{\text{barge}} + \underbrace{\frac{\left(V_{\text{river}}^2 - cV_{\text{wind}}^2\right)}{(1-c)}}_{=C} = 0, \tag{9}$$

$$V_{\text{barge}} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}. \tag{10}$$

Using the give data,

ρ_{air} = 1.23 kg/m^3,
V_{river} = 1 m/s,
V_{wind} = 10 m/s,
=> $c = 1.41*10^{-3}$, $A = 1$, B = -2.03 m/s, $C = 8.60*10^{-1}$ m^2/s^2
=> V_{barge} = 1.43 m/s, 0.601 m/s.

Note that it's not possible for the barge to move faster than the river's speed of 1 m/s, so $V_{\text{barge}} \neq 1.43$ m/s. Thus, the correct answer is V_{barge} = 0.601 m/s (downstream).

If we had assumed that V_{barge} was moving upstream (same direction as V_{wind}), then Eq. (5) would be,

$$\sum F_x = 0 = c_{D,\text{below}} \tfrac{1}{2}\rho_{H_2O}\left(V_{\text{river}} + V_{\text{barge}}\right)^2 A_{\substack{\text{wetted,}\\\text{below}}} - c_{D,\text{above}} \tfrac{1}{2}\rho_{\text{air}}\left(V_{\text{wind}} - V_{\text{barge}}\right)^2 A_{\substack{\text{wetted,}\\\text{above}}}, \tag{11}$$

which would simplify to,

$$V_{\text{barge}}^2 + \frac{2\left(V_{\text{river}} + cV_{\text{wind}}\right)}{(1-c)}V_{\text{barge}} + \frac{\left(V_{\text{river}}^2 - cV_{\text{wind}}^2\right)}{(1-c)} = 0. \tag{12}$$

Solving this equation gives,

V_{barge} = -0.601 m/s, -1.43 m/s.

Thus, we see that the original choice of direction for V_{barge} (upstream) was incorrect and the barge is actually moving downstream. As in the previous discussion, the barge cannot move faster than the river speed so the correct speed is 0.601 m/s.