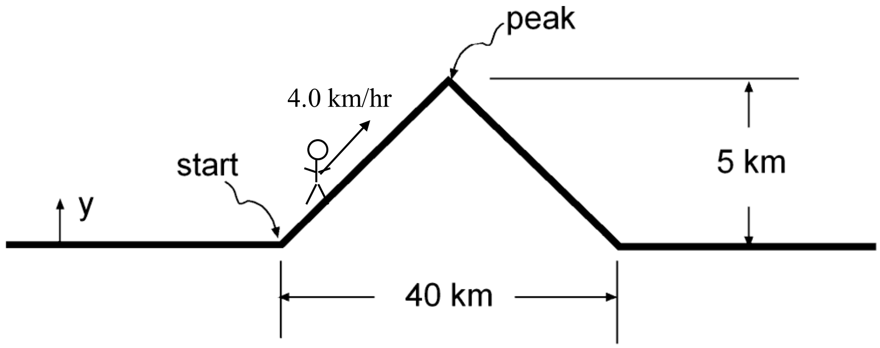
You are climbing up the side of Triangle Mountain, so named because the sides are relatively straight, making the cross-section of the mountain look like a triangle. The mountain is 5 km high and has a base of 40 km, as shown.



You are worried about how hot you will get on your trip and the rate at which the temperature will change with time. The temperature decreases with altitude at a rate of 5 °C/km. Also, the temperature changes in time as the sun heats the ground. The temperature (in °C) as you climb the mountain is given by:

$$T(y,t) = 25\ \text{C} - (0.005\ \ \text{C/m})\,y + (5\ \ \text{C})\sin\left[\frac{2\pi(t-t_o)}{24\text{ hrs}}\right]$$

where t is measured in hours from midnight, $t_o = 9$ hrs, and y is the altitude measured in meters from the base of the mountain.

You start ascending the mountain at 6:00 A.M. and travel at a speed of 4.0 km/hr up the mountain side.

a. Derive an expression for the time derivative of temperature you experience as you climb up the mountain.
b. Calculate the rate of change in temperature at the moment you reach the mountain peak (in °C/hr).

SOLUTION:

Write the Lagrangian time derivative, keeping in mind that the two variables of interest are *t* and *y*:

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + u_y \frac{\partial T}{\partial y} \tag{1}$$

where

$$\frac{\partial T}{\partial t} = 5\ \text{C} \cdot \frac{2\pi}{24\ \text{hrs}} \cos\left[\frac{2\pi\left(t - t_0\right)}{24\ \text{hrs}}\right], \tag{2}$$

$$u_y = \left(4.0\ \text{km/hr}\right)\left[\frac{5\ \text{km}}{\sqrt{\left(20\ \text{km}\right)^2 + \left(5\ \text{km}\right)^2}}\right] = 0.97\ \text{km/hr}, \tag{3}$$

$$\frac{\partial T}{\partial y} = -0.005\ \ \text{C/m}. \tag{4}$$

Substitute and simplify.

$$\boxed{\frac{DT}{Dt} = \frac{5\pi\ \text{C}}{12\ \text{hrs}} \cos\left[\frac{2\pi\left(t - t_0\right)}{24\ \text{hrs}}\right] - \left(4.85\ \ \text{C/hr}\right)} \tag{5}$$

You reach the peak in 5.15 hrs (= 5 km / (0.97 km/hr)), which means you reach the peak at 11:09 A.M (t = 11.15 hrs). Evaluating Eqn. (5) using t_0 = 9 hrs gives, at the moment you reach the peak:

$$\boxed{DT/Dt = -3.74\ °\text{C/hr}} \tag{6}$$