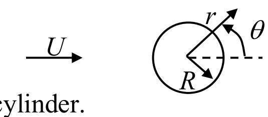
The velocity field for a steady, inviscid flow over a circular cylinder of radius R is given by:

$$\mathbf{u} = U\cos\theta\left[1-\left(\frac{R}{r}\right)^2\right]\hat{\mathbf{e}}_r - U\sin\theta\left[1+\left(\frac{R}{r}\right)^2\right]\hat{\mathbf{e}}_\theta$$



where U is the (constant) speed of the flow far upstream of the cylinder.

a. Determine the acceleration of a fluid particle moving along the stagnation streamline ($\theta = \pi$).

b. Determine the acceleration of a fluid particle moving along the cylinder surface ($r = R$).

SOLUTION:

The acceleration of a fluid particle may be found by taking the Lagrangian derivative of the velocity.

$$\mathbf{a} = \frac{D\mathbf{u}}{Dt} = \left(\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta^2}{r} \right)\hat{\mathbf{e}}_r + \left(\frac{\partial u_\theta}{\partial t} + u_r \frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r u_\theta}{r} \right)\hat{\mathbf{e}}_\theta \quad (1)$$

where

$$u_r = U\cos\theta\left[1 - \left(\frac{R}{r}\right)^2\right] \quad (2)$$

$$\frac{\partial u_r}{\partial t} = 0 \quad (3)$$

$$\frac{\partial u_r}{\partial r} = 2U\cos\theta\frac{R^2}{r^3} \quad (4)$$

$$\frac{\partial u_r}{\partial \theta} = -U\sin\theta\left[1 - \left(\frac{R}{r}\right)^2\right] \quad (5)$$

$$u_\theta = -U\sin\theta\left[1 + \left(\frac{R}{r}\right)^2\right] \quad (6)$$

$$\frac{\partial u_\theta}{\partial t} = 0 \quad (7)$$

$$\frac{\partial u_\theta}{\partial r} = 2U\sin\theta\frac{R^2}{r^3} \quad (8)$$

$$\frac{\partial u_\theta}{\partial \theta} = -U\cos\theta\left[1 + \left(\frac{R}{r}\right)^2\right] \quad (9)$$

Along the stagnation streamline ($\theta = \pi$, $r \geq R$):

$$u_r\big|_{\theta=\pi} = -U\left[1 - \left(\frac{R}{r}\right)^2\right] \quad (10)$$

$$\left.\frac{\partial u_r}{\partial r}\right|_{\theta=\pi} = -2U\frac{R^2}{r^3} \quad (11)$$

$$\left.\frac{\partial u_r}{\partial \theta}\right|_{\theta=\pi} = 0 \quad (12)$$

$$u_\theta\big|_{\theta=\pi} = 0 \quad (13)$$

$$\left.\frac{\partial u_\theta}{\partial r}\right|_{\theta=\pi} = 0 \quad (14)$$

$$\left.\frac{\partial u_\theta}{\partial \theta}\right|_{\theta=\pi} = U\left[1 + \left(\frac{R}{r}\right)^2\right] \quad (15)$$

Substitute Eqns. (10) - (15) into Eqn. (1) and simplify.

$$\boxed{\mathbf{a}\big|_{\theta=\pi} = 2U^2\frac{R^2}{r^3}\left[1 - \left(\frac{R}{r}\right)^2\right]\hat{\mathbf{e}}_r} \quad (16)$$

Along the cylinder's surface ($r = R$):

$$u_r\big|_{r=R} = 0 \tag{17}$$

$$\left.\frac{\partial u_r}{\partial r}\right|_{r=R} = \frac{2U\cos\theta}{R} \tag{18}$$

$$\left.\frac{\partial u_r}{\partial \theta}\right|_{r=R} = 0 \tag{19}$$

$$u_\theta\big|_{r=R} = -2U\sin\theta \tag{20}$$

$$\left.\frac{\partial u_\theta}{\partial r}\right|_{r=R} = \frac{2U\sin\theta}{R} \tag{21}$$

$$\left.\frac{\partial u_\theta}{\partial \theta}\right|_{r=R} = -2U\cos\theta \tag{22}$$

Substitute Eqns. (10) - (15) into Eqn. (1) and simplify.

$$\boxed{\mathbf{a}\big|_{r=R} = \left(-\frac{4U^2\sin^2\theta}{R}\right)\hat{\mathbf{e}}_r + \left(\frac{4U^2\sin\theta\cos\theta}{R}\right)\hat{\mathbf{e}}_\theta} \tag{23}$$