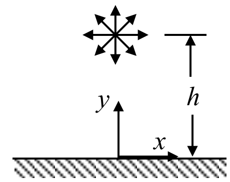
The velocity field for a plane source located a distance, *h*, above an infinite wall aligned along the *x* axis is given by:

$$\mathbf{u} = \frac{q}{2\pi\left[x^2 + (y-h)^2\right]}\left[x\hat{\mathbf{i}} + (y-h)\hat{\mathbf{j}}\right] + \frac{q}{2\pi\left[x^2 + (y+h)^2\right]}\left[x\hat{\mathbf{i}} + (y+h)\hat{\mathbf{j}}\right]$$



where q is the source strength.

1. Derive expressions for the velocity and acceleration for a fluid particle that moves along the wall.
2. Verify that the velocity and acceleration normal to the wall are zero.

SOLUTION:

The velocity of a particle moving along the wall ($y = 0$) is:

$$\mathbf{u} = \frac{q}{2\pi\left[x^2+h^2\right]}\left[x\hat{\mathbf{i}} - h\hat{\mathbf{j}}\right] + \frac{q}{2\pi\left[x^2+h^2\right]}\left[x\hat{\mathbf{i}} + h\hat{\mathbf{j}}\right]$$

$$\boxed{\mathbf{u} = \frac{qx\hat{\mathbf{i}}}{\pi\left[x^2+h^2\right]}} \tag{1}$$

The acceleration of a fluid particle is determined by taking the Lagrangian derivative of the velocity.

$$\mathbf{a} = \frac{D\mathbf{u}}{Dt} = \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} = \frac{\partial \mathbf{u}}{\partial t} + u_x\frac{\partial \mathbf{u}}{\partial x} + u_y\frac{\partial \mathbf{u}}{\partial y}$$

where

$$\frac{\partial \mathbf{u}}{\partial t} = \mathbf{0}$$

$$\frac{\partial u_x}{\partial x} = \frac{\partial}{\partial x}\left\{\frac{qx}{2\pi\left[x^2+(y-h)^2\right]} + \frac{qx}{2\pi\left[x^2+(y+h)^2\right]}\right\}$$

$$= \frac{q}{2\pi}\left\{\frac{1}{\left[x^2+(y-h)^2\right]} + \frac{-2x^2}{\left[x^2+(y-h)^2\right]^2} + \frac{1}{\left[x^2+(y+h)^2\right]} + \frac{-2x^2}{\left[x^2+(y+h)^2\right]^2}\right\}$$

$$\frac{\partial u_x}{\partial y} = \frac{\partial}{\partial y}\left\{\frac{qx}{2\pi\left[x^2+(y-h)^2\right]} + \frac{qx}{2\pi\left[x^2+(y+h)^2\right]}\right\}$$

$$= \frac{q}{2\pi}\left\{\frac{-2x(y-h)}{\left[x^2+(y-h)^2\right]^2} + \frac{-2x(y+h)}{\left[x^2+(y+h)^2\right]^2}\right\}$$

$$\frac{\partial u_y}{\partial x} = \frac{\partial}{\partial x}\left\{\frac{q(y-h)}{2\pi\left[x^2+(y-h)^2\right]} + \frac{q(y+h)}{2\pi\left[x^2+(y+h)^2\right]}\right\}$$

$$= \frac{q}{2\pi}\left\{\frac{-2x(y-h)}{\left[x^2+(y-h)^2\right]^2} + \frac{-2x(y+h)}{\left[x^2+(y+h)^2\right]^2}\right\}$$

$$\frac{\partial u_y}{\partial y} = \frac{\partial}{\partial y}\left\{\frac{q(y-h)}{2\pi\left[x^2+(y-h)^2\right]} + \frac{q(y+h)}{2\pi\left[x^2+(y+h)^2\right]}\right\}$$

$$= \frac{q}{2\pi}\left\{\frac{1}{\left[x^2+(y-h)^2\right]} + \frac{-2(y-h)^2}{\left[x^2+(y-h)^2\right]^2} + \frac{1}{\left[x^2+(y+h)^2\right]} + \frac{-2(y+h)^2}{\left[x^2+(y+h)^2\right]^2}\right\}$$

For a particle that moves along the wall ($y = 0$):

$$u_x = \frac{2xq}{2\pi\left[x^2+h^2\right]}$$

$$u_y = 0$$

$$\frac{\partial u_x}{\partial x} = \frac{q}{2\pi}\left\{\frac{2}{\left[x^2+h^2\right]}+\frac{-4x^2}{\left[x^2+h^2\right]^2}\right\}$$

$$\frac{\partial u_x}{\partial y} = 0$$

$$\frac{\partial u_y}{\partial x} = 0$$

$$\frac{\partial u_y}{\partial y} = \frac{q}{2\pi}\left\{\frac{2}{\left[x^2+h^2\right]}+\frac{-4h^2}{\left[x^2+h^2\right]^2}\right\}$$

$$\boxed{\therefore \mathbf{a} = \left[\frac{xq}{\pi\left(x^2+h^2\right)}\right]\left\{\frac{q}{2\pi}\left[\frac{2}{\left(x^2+h^2\right)}+\frac{-4x^2}{\left(x^2+h^2\right)^2}\right]\right\}\hat{\mathbf{i}}} \quad (2)$$

Eqns. (1) and (2) have no y-components indicating that the velocity and acceleration normal to the wall (in the y-direction) are zero.