

The market price,  $P$  (in dollars), of used cars of a certain model is found to be:

$$P = \$1000 + (\$0.02 / \text{mile})x - (\$2 / \text{day})t$$

where  $x$  is the distance in miles west of Detroit, MI and  $t$  is the time in days. If a car of this model is driven from Detroit at  $t=0$  towards the west at a rate of 400 miles per day, determine:

- a. whether the value of the car is increasing or decreasing, and
- b. how much of this change is due to depreciation and how much is due to moving into a better market.

SOLUTION:

To determine if the value of the car is decreasing, take the Lagrangian derivative of the market price.

$$\frac{DP}{Dt} = \frac{\partial P}{\partial t} + u \frac{\partial P}{\partial x} = (-\$2/\text{day}) + (400 \text{ miles/day})(\$0.02/\text{mile}) \quad (\text{where } u \text{ is the speed of the car}) \quad (1)$$

= \$8/day

$$\therefore \frac{DP}{Dt} = \$6/\text{day} \quad \text{Hence, the value of the car is increasing.}$$

The car depreciates at a rate of -\$2/day (this is  $\partial P/\partial t$ ). The change in the car's value increases at a rate of \$8/day due to moving into a different market (this is  $u\partial P/\partial x$ ).