A supersonic wind tunnel test section is designed to have a Mach number of 2.5 at a temperature of 60 °F and 5 psia. The fluid is air.

a. Determine the required inlet stagnation temperature and pressure.

b. Calculate the required mass flow rate for a test section area of 2.0 ft^2.

SOLUTION:

The stagnation properties may be found using the isentropic relations:

$$\frac{p_{TS}}{p_0} = \left(1 + \frac{\gamma - 1}{2}\mathrm{Ma}_{\mathrm{TS}}^2\right)^{\frac{\gamma}{1-\gamma}} \tag{1}$$

$$\frac{T_{TS}}{T_0} = \left(1 + \frac{\gamma - 1}{2}\mathrm{Ma}_{\mathrm{TS}}^2\right)^{-1} \tag{2}$$

where

p_{TS} = 5 psia = 720 $\mathrm{lb_f/ft^2}$
T_{TS} = (60 + 459) °R = 519 °R
Ma_{TS} = 2.5
γ_{air} = 1.4
$\therefore \boxed{p_0 = 85.4 \text{ psia}}$ and $\boxed{T_0 = 1170 \text{ °R}}$

The mass flow rate may be found using:

$$\dot{m}_{TS} = \rho_{TS} V_{TS} A_{TS} = \left(\frac{p_{TS}}{RT_{TS}}\right)\left(c_{TS}\mathrm{Ma}_{TS}\right) A_{TS} \tag{3}$$

where the speed of sound in the test section, c_{TS}, is:

$$c_{TS} = \sqrt{\gamma R T_{TS}} \tag{4}$$

Using the given data:

R_{air} = 53.3 $(\mathrm{ft \cdot lb_f})/(\mathrm{lb_m \cdot °R})$
A_{TS} = 2 $\mathrm{ft^2}$
$\Rightarrow c_{TS}$ = 1120 ft/s
ρ_{TS} = 0.0260 $\mathrm{lb_m/ft^3}$
$\boxed{\dot{m}_{TS} = 145 \text{ lb}_\mathrm{m}/\mathrm{s}}$