Natural gas, with the thermodynamic properties of methane, flows in an insulated, underground pipeline of 0.6 m diameter. The gage pressure at the inlet to a compressor station is 0.5 MPa; the outlet pressure is 8.0 MPa (gage). The gas temperature and speed at inlet are 13 °C and 32 m/s, respectively. The compressor's efficiency is 85%.

a. Calculate the mass flow rate of natural gas through the pipeline.
b. Evaluate the gas temperature and speed at the compressor outlet.
c. Calculate the power required to drive the compressor.

SOLUTION:

First determine the mass flow rate.

$$\dot{m} = \rho_i V_i A_i = \frac{p_i}{RT_i} V_i \tfrac{\pi}{4} D_i^2 \tag{1}$$

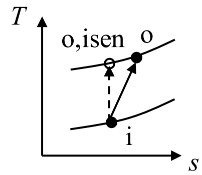
where the subscript "i" refers to the inlet conditions and

p_i = (0.5 MPa + 101 kPa) = 6.0*10^5 Pa
R = 518.3 J/(kg·K)
T_i = (13 + 273) K = 286 K
V_i = 32 m/s
D_i = 0.6 m

$$\boxed{\therefore \dot{m} = 3.7*10^1 \text{ kg/s}}$$

The temperature at the exit may be found by applying the isentropic relations for a perfect gas across the compressor and making use of the compressor's efficiency. Note that for methane, the specific heat ratio is $\gamma = 1.31$.

$$\frac{T_{o,\text{isen}}}{T_i} = \left(\frac{p_o}{p_i}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_{o,\text{isen}} = T_i \left(\frac{p_o}{p_i}\right)^{\frac{\gamma-1}{\gamma}} \tag{2}$$



The term $T_{o,\text{isen}}$ assumes an isentropic process through the compressor. However, we know that the compressor isn't 100% efficient so, from the definition of the efficiency,

$$\eta_C = \frac{\dot{W}_{\text{on gas,isen}}}{\dot{W}_{\text{on gas,actual}}}, \tag{3}$$

where the power acting on the gas is found from the 1st Law,

$$\dot{m}_o\left(h + \tfrac{1}{2}V^2\right)_o - \dot{m}_i\left(h + \tfrac{1}{2}V^2\right)_i = \dot{Q}_{\substack{\text{into}\\\text{gas}}} + \dot{W}_{\text{on gas}}, \tag{4}$$

$$\dot{W}_{\text{on gas}} = \dot{m}\left[\left(h_o - h_i\right) + \tfrac{1}{2}\left(V_o^2 - V_i^2\right)\right] \text{ (assuming adiabatic operation and } \dot{m} = \dot{m}_i = \dot{m}_o \text{ from COM)}, \tag{5}$$

$$\dot{W}_{\text{on gas}} = \dot{m}\left[c_P\left(T_o - T_i\right) + \tfrac{1}{2}\left(V_o^2 - V_i^2\right)\right], \tag{6}$$

where the last expression is specifically for a perfect gas. Substituting into Eq. (3) and simplifying gives,

$$\eta_C = \frac{\dot{m}\left[c_P\left(T_{o,\text{isen}} - T_i\right) + \tfrac{1}{2}\left(V_{o,\text{isen}}^2 - V_i^2\right)\right]}{\dot{m}\left[c_P\left(T_{o,\text{actual}} - T_i\right) + \tfrac{1}{2}\left(V_{o,\text{actual}}^2 - V_i^2\right)\right]} = \frac{c_P\left(T_{o,\text{isen}} - T_i\right) + \tfrac{1}{2}\left(V_{o,\text{isen}}^2 - V_i^2\right)}{c_P\left(T_{o,\text{actual}} - T_i\right) + \tfrac{1}{2}\left(V_{o,\text{actual}}^2 - V_i^2\right)}. \tag{7}$$

Note that the change in the specific kinetic energy is generally much smaller than the change in the specific enthalpy so that the efficiency becomes,

$$\eta_C \approx \frac{c_P\left(T_{o,\text{isen}} - T_i\right)}{c_P\left(T_{o,\text{actual}} - T_i\right)} = \frac{T_{o,\text{isen}} - T_i}{T_{o,\text{actual}} - T_i}, \tag{8}$$

$$T_{o,\text{actual}} = T_i + \frac{T_{o,\text{isen}} - T_i}{\eta_C}. \tag{9}$$

Using the given data along with p_o = (8.0 MPa + 101 kPa) = 8.1*10^6 Pa (abs),

$T_{o,\text{isen}} = 529$ K

$\boxed{T_o = 573\text{ K}}$

Note that the subscript "$_{\text{actual}}$" has been dropped for convenience.

The speed of the gas at the outlet may be found by applying conservation of mass across the compressor.

$$\dot{m}_o = \dot{m}_i \Rightarrow \rho_o V_o A_o = \rho_i V_i A_i \Rightarrow V_o = V_i \frac{\rho_i}{\rho_o} = V_i \frac{\left(p_i \big/ RT_i \right)}{\left(p_o \big/ RT_o \right)} = V_i \left(\frac{p_i}{p_o} \right)\left(\frac{T_o}{T_i} \right) \tag{10}$$

$\boxed{\therefore V_o = 4.7\text{ m/s}}$

Note that,

$$|c_P \Delta T| >> |\tfrac{1}{2}\Delta V^2|, \tag{11}$$

where,

$$c_p = \frac{\gamma}{\gamma - 1} R \Rightarrow c_P = 2190\text{ J/(kg}\cdot\text{K)}, \tag{12}$$

so that the assumption in Eq. (8) is a good one.

The power required to drive the compressor may be found by applying conservation of energy across the compressor (Eq. (6)). Substituting the (non-isentropic) values gives:

$\boxed{\dot{W}_{\text{on gas}} = 23\text{ MW}}$