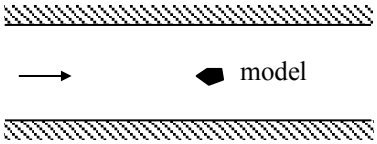
In wind-tunnel testing near Ma = 1, a small area decrease caused by model blockage can be important. Suppose the test section area is 1 m^2, with unblocked test conditions Ma = 1.10 and T = 20 °C.

a. What model area will first cause the test section to choke?
b. If the model cross section is 0.004 m^2 (0.4 % blockage), what percentage change in test section velocity results?

SOLUTION:



First determine the area when the test section will choke. This area will be the sonic area.

$$\frac{A}{A^*} = \frac{1}{\mathrm{Ma}}\left(\frac{1+\frac{\gamma-1}{2}\mathrm{Ma}^2}{1+\frac{\gamma-1}{2}}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \tag{1}$$

Using $A_{TS} = 1$ m^2, Ma = 1.10, and $\gamma_{air} = 1.4$, $A^* = 0.992$ m^2. Thus, the model area that will cause the test section to choke is $A_{model} = A_{TS} - A^* = (1 - 0.992)$ m^2 = 0.008 m^2.

Using Eqn. (1) with $A = (1 - 0.004)$ m^2 = 0.996 m^2 and $A^* = 0.992$ m^2, the Mach number in the test section with the blockage is Ma = 1.07.

The velocity corresponding to a given Mach number is given by:

$$V = c\mathrm{Ma} = \sqrt{\gamma RT}\,\mathrm{Ma} \tag{2}$$

where the local temperature is found using:

$$T = T_0\left(1+\frac{\gamma-1}{2}\mathrm{Ma}^2\right)^{-1} \tag{3}$$

The percent change in the test section velocity is:

$$\%\ \text{change} = \frac{V_{\text{w/ blockage}} - V_{\text{w/o blockage}}}{V_{\text{w/o blockage}}} = \frac{V_{\text{w/ blockage}}}{V_{\text{w/o blockage}}} - 1$$

$$= \frac{\mathrm{Ma}_{\text{w/ blockage}}}{\mathrm{Ma}_{\text{w/o blockage}}}\sqrt{\frac{T_{\text{w/ blockage}}}{T_{\text{w/ blockage}}}} - 1$$

$$\therefore\ \%\ \text{change} = \frac{\mathrm{Ma}_{\text{w/ blockage}}}{\mathrm{Ma}_{\text{w/o blockage}}}\left(\frac{1+\frac{\gamma-1}{2}\mathrm{Ma}^2_{\text{w/ blockage}}}{1+\frac{\gamma-1}{2}\mathrm{Ma}^2_{\text{w/o blockage}}}\right)^{-\frac{1}{2}} - 1 \tag{4}$$

Using $\mathrm{Ma}_{\text{w/ blockage}} = 1.07$, $\mathrm{Ma}_{\text{w/o blockage}} = 1.10$, and $\gamma_{air} = 1.4$, % change = -2.2%.