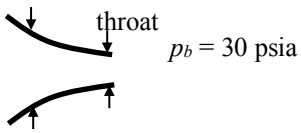
Air flows isentropically through a converging nozzle. At a section where the nozzle area is 0.013 ft^2, the local pressure, temperature, and Mach number are 60 psia, 40 °F, and 0.52, respectively. The back pressure is 30 psia. Determine:

a. the Mach number at the throat,
b. the mass flow rate, and
c. the throat area.

SOLUTION:



$A = 0.013$ ft^2
$p = 60$ psia
$T = 40$ °F = 500 °R
Ma = 0.52

First determine if the flow is choked by checking the pressure ratio at the exit.

$$\frac{p}{p_0} = \left(1 + \frac{\gamma-1}{2}\text{Ma}^2\right)^{\frac{\gamma}{1-\gamma}} \Rightarrow p_0 = 72.15 \text{ psia} \quad (1)$$

using $p = 60$ psia, $\gamma = 1.4$, and Ma = 0.52.

$$\frac{p_b}{p_0} = \frac{30 \text{ psia}}{72.15 \text{ psia}} = 0.4158 < \frac{p^*}{p_0} = 0.5283 \Rightarrow \text{The flow is choked!} \quad (2)$$

Since the flow is choked, $\boxed{\text{Ma}_T = 1}$ and the throat area will equal the sonic area:

$$\frac{A}{A_T} = \frac{A}{A^*} = \frac{1}{\text{Ma}}\left(\frac{1 + \frac{\gamma-1}{2}\text{Ma}^2}{1 + \frac{\gamma-1}{2}}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \Rightarrow \boxed{A_T = A^* = 9.97*10^{-3} \text{ ft}^2} \quad (3)$$

where $A = 0.013$ ft^2, $\gamma = 1.4$, and Ma = 0.52.

The mass flow rate will be the choked mass flow rate:

$$\dot{m}_{\text{choked}} = \left(1 + \frac{\gamma-1}{2}\right)^{\frac{1+\gamma}{2(1-\gamma)}} p_0 \sqrt{\frac{\gamma}{RT_0}} A^* \Rightarrow \boxed{\dot{m}_{\text{choked}} = 2.40 \text{ lb}_\text{m}/\text{s}} \quad (4)$$

where $\gamma = 1.4$, $R = 53.3$ (lb$_\text{f}$·ft)/(lb$_\text{m}$·°R), $p_0 = 72.15$ psia = $1.04*10^4$ lb$_\text{f}$/ft^2, $A^* = 9.97*10^{-3}$ ft^2 and

$$\frac{T}{T_0} = \left(1 + \frac{\gamma-1}{2}\text{Ma}^2\right)^{-1} \Rightarrow \underline{T_0 = 527 \text{ °R}} \text{ (Ma = 0.52 and } T = 500 \text{ °R)} \quad (5)$$