The velocity for a steady, incompressible flow in the xy plane is given by,

$$\mathbf{u} = \frac{A}{x}\hat{\mathbf{i}} + \frac{By}{x}\hat{\mathbf{j}},$$

where $A = 2\ \text{m}^2/\text{s}$, $B = 1$ m/s, and the coordinates x and y are measured in meters.

a. Obtain an equation for the streamline that passes through the point $(x, y) = (1\ \text{m}, 3\ \text{m})$.

b. Calculate the time required for a fluid particle to move from $x = 1$ m to $x = 2$ m in this flow field.

SOLUTION:

The slope of the streamline is given by,

$$\frac{dy}{dx}=\frac{u_y}{u_x}=\frac{B(y/x)}{A(1/x)}=\frac{B}{A}y\,. \qquad (1)$$

Solving this differential equation gives,

$$\frac{dy}{dx}=\frac{B}{A}y\Rightarrow\int_{y=y_0}^{y=y}\frac{dy}{y}=\frac{B}{A}\int_{x=x_0}^{x=x}dx\Rightarrow\ln\left(\frac{y}{y_0}\right)=\frac{B}{A}(x-x_0)\Rightarrow y=y_0\exp\left[\frac{B}{A}(x-x_0)\right], \qquad (2)$$

$$\boxed{y=(3\text{ m})\exp\left[\left(\frac{1}{2}\text{ m}^{-1}\right)(x-1\text{ m})\right]}$$ using (x_0, y_0) = (1 m, 3 m). (3)

To determine the time to travel from x_1 to x_2,

$$u_x=\frac{A}{x}\Rightarrow\frac{dx}{dt}=\frac{A}{x}\Rightarrow\int_{x=x_1}^{x=x_2}x\,dx=\int_{t=t_1}^{t=t_2}A\,dt\Rightarrow\tfrac{1}{2}\left(x_2^2-x_1^2\right)=A\left(t_2-t_1\right), \qquad (4)$$

$$\boxed{\Delta t=\frac{1}{2A}\left(x_2^2-x_1^2\right)}. \qquad (5)$$

Using the given data,

A = 2 m^2/s

x_1 = 1 m,

x_2 = 2 m,

=> $\boxed{\Delta t = 0.75\text{ s}}$