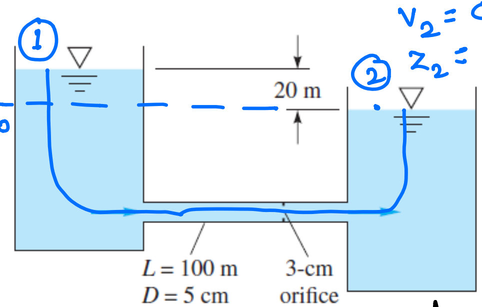
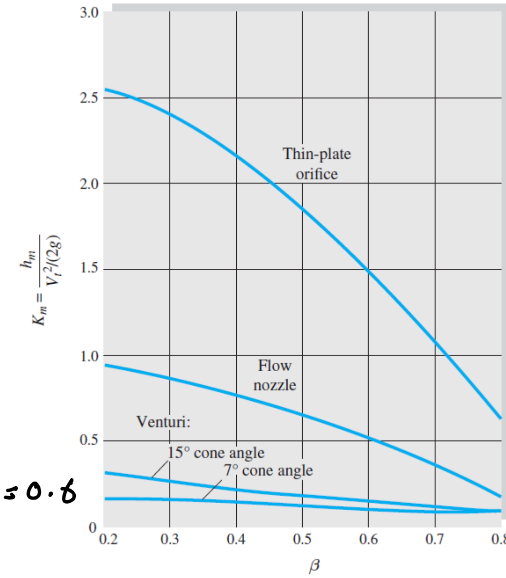
A pipe connecting two reservoirs, as in Fig. P6.146, contains a thin-plate orifice. For water flow at 20°C, estimate (*a*) the volume flow through the pipe and (*b*) the pressure drop across the orifice plate.

$p_1 = 0$, $V_1 = 0$, $z_1 = 20\,m$

$p_2 = 0$, $V_2 = 0$, $z_2 = 0$



P6.146



$\beta = \frac{d}{D} = \frac{3}{5} = 0.6$

$T = 20°C$

$\rho = 998\ kg/m^3$

$\mu = 0.001\ kg/m\,s.$

Apply EBE betn ① & ②

$$\left(\frac{p}{\rho g} + \alpha\frac{V^2}{2g} + z\right)_2 = \left(\frac{p}{\rho g} + \alpha\frac{V^2}{2g} + z\right)_1 - H_L + H_S$$

$H_S = 0$

$$\Rightarrow z_1 - z_2 = H_L.$$

$z_1 - z_2 = 20\,m$

$$H_L = \sum K_i \frac{V_i^2}{2g}$$

① Major loss → long section 100 m.

② Minor losses.
↳ Entrance
↳ Exit.

③ Orific loss.

$$H_L = K_{Major}\frac{V^2}{2g} + K_{ent}\frac{V^2}{2g} + K_{exit}\frac{V^2}{2g} + K_{orific}\frac{V^2}{2g}$$

$K_{Major} = f\frac{L}{D}$, $K_{ent} = 0.5$, $K_{exit} = 1$, $K_{orific} = 1.5$ ($\beta = 0.6$)

$$H_L = \frac{V^2}{2g}\left(f\frac{L}{D} + 0.5 + 1 + 1.5\right) \quad \text{— ①}$$

$$\therefore \frac{V^2}{2g}\left(f \frac{100\ m}{0.05\ m} + 3 \right) = z_1 - z_2 = 20\ m.$$

$\frac{100\ m}{0.05\ m} \to 2000$

$g = 9.81\ m/s^2$

$$\Rightarrow V^2 = \frac{2 \times 9.81 \times 20}{f\,2000 + 3}$$

Re ~ unknown — (2)

Guess $Re = 5 \times 10^5 \rightarrow f \simeq 0.02$

Estimated $V = 3.02\ m/s$

$$\text{Estimated } Re = \frac{\rho V D}{\mu} = \frac{998 \times 3.02 \times 0.05}{0.001}$$

$$\simeq 1.5 \times 10^5$$

Estimated Re (1.5×10^5) < Guess Re. (5×10^5)

Improved guess $Re = 1.5 \times 10^5$

$f = 0.016$

Substitute in (2)

Estimated $V \approx 3.33\ m/s$.

$$\text{estimated } Re \approx \frac{\rho V D}{\mu} = \frac{998 \times 3.33 \times 0.05}{0.001} \approx 1.6 \times 10^5$$

Next iteration Guess $Re = 1.6 \times 10^5$

↓

Converge $f \simeq 0.0162$.

$V \simeq 3.33\ m/s$.

$Re \simeq 1.66 \times 10^5$

$\beta = \frac{d}{D} = 0.6$

Solution

Cd for orifice ←

$$Q = VA = \frac{\pi}{4} D^2 V = \frac{\pi}{4} \times (0.05)^2 \, 3.33$$

Ans. to part (a)

$$\boxed{Q \approx 0.00653 \ m^3/s.}$$

b) $Re_D = 166000$ [Assume D : ½D taps]

$\beta = 0.6$

$C_d \approx 0.609 \Leftarrow$ C_d vs Re_D for orifice.

$$Q = C_d A_t \sqrt{\frac{2(\Delta P_{orifice})}{\rho(1-\beta^4)}}$$

$A_t = \frac{\pi}{4}(0.03)^2$

↓ Solve for $\Delta P_{orifice}$

$$\boxed{\Delta P_{orifice} \approx 100 \ kPa.}$$