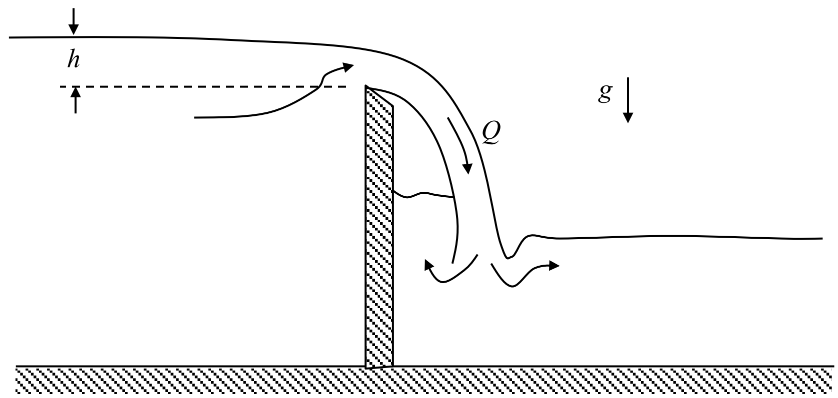
A 1/16th-scale model of a weir has a measured flow rate of $Q = 2.1\ \text{ft}^3/\text{s}$ when the upstream water height is $h = 6.3$ in. The flow rate is known to be a function of the acceleration due to gravity, g, the weir width (into the page), b, and the upstream water height, h. Furthermore, the flow rate is found to be directly proportional to the weir width, b. What is the flow rate over the prototype weir when the upstream water height is $h = 3.2$ ft.



SOLUTION:

1. Write the dimensional functional relationship.

$$Q = f_1(g, h, b)$$

2. Determine the basic dimensions of each parameter.

$$[Q] = L^3/T$$
$$[g] = L/T^2$$
$$[h] = L$$
$$[b] = L$$

3. Determine the number of Π terms required to describe the functional relationship.
   # of variables = 4 (Q, g, h, b)
   # of reference dimensions = 2 (L, T)

   (# Π terms) = (# of variables) – (# of reference dimensions) = 4 – 2 = 2

4. Choose three repeating variables by which all other variables will be normalized (same # as the # of reference dimensions).
   g, h

5. Make the remaining non-repeating variables dimensionless using the repeating variables.

$$\Pi_1 = \frac{Q}{\sqrt{gh^5}} \qquad \text{(by inspection)}$$

$$\Pi_2 = \frac{b}{h} \qquad \text{(by inspection)}$$

6. Verify that each Π term is, in fact, dimensionless.

$$[\Pi_1] = \left[\frac{Q}{\sqrt{gh^5}}\right] = \frac{L^3/T}{\sqrt{L/T^2 \cdot L^5}} = 1 \quad \text{OK!}$$

$$[\Pi_2] = \left[\frac{b}{h}\right] = \frac{L}{L} = 1$$

7. Re-write the original relationship in dimensionless terms.

$$\boxed{\frac{Q}{\sqrt{gh^5}} = f_2\left(\frac{b}{h}\right)} \tag{1}$$

We are also told that $Q \propto b$ so that Eqn. (1) becomes:

$$\frac{Q}{\sqrt{gh^5}} = c\left(\frac{b}{h}\right) \tag{2}$$

$$\therefore \frac{Q}{b\sqrt{gh^3}} = c \tag{3}$$

where c is a constant of proportionality.

Since the right-hand side of Eq. (1) is a constant, then:

$$\left(\frac{Q}{b\sqrt{gh^3}}\right)_{\text{prototype}} = \left(\frac{Q}{b\sqrt{gh^3}}\right)_{\text{model}}$$

$$Q_{\text{prototype}} = Q_{\text{model}} \frac{\left(b\sqrt{gh^3}\right)_{\text{prototype}}}{\left(b\sqrt{gh^3}\right)_{\text{model}}}$$

The gravitational acceleration is the same for the model and prototype (i.e., $g_1 = g_2$):

$$\boxed{\therefore Q_{\text{prototype}} = Q_{\text{model}}\left(\frac{b_{\text{prototype}}}{b_{\text{model}}}\right)\left(\frac{h_{\text{prototype}}}{h_{\text{model}}}\right)^{3/2}} \tag{4}$$

Use the given data to determine Q_2.

Q_{model} = 2.1 ft^3/s
h_{model} = 6.3 in. = 0.525 ft.
$b_{\text{model}}/b_{\text{prototype}}$ = 1/16
$h_{\text{prototype}}$ = 3.2 ft
$\Rightarrow$ $\boxed{Q_{\text{prototype}} = 506 \text{ ft}^3\text{/s}}$