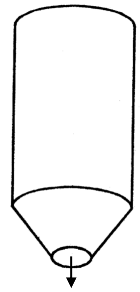
Hoppers are a commonly used device in the handling and storage of particulate materials. A hopper design typically consists of a bin section located above a converging section with a hole located in the bottom through which the particulate material flows (refer to the figures below).



One interesting observation with hopper flows is that the mass flow rate from the hopper exit is independent of the height of the material above the exit and the bin diameter (except when the hopper is nearly empty). The parameters that do affect the discharge rate (assuming cohesionless particles) include the hopper exit diameter, the acceleration due to gravity, the angle of the hopper walls, the friction coefficient between the particulate material and the walls and between the particles themselves, and the bulk density of the material at the discharge plane.

a. Perform a dimensional analysis to determine the dimensionless quantities that govern flow from a hopper.
b. If the same hopper and particulate material are used (i.e., the wall angle and friction properties remain the same), how will the mass flow rate from the hopper change if the hopper exit diameter is doubled?
c. Compare the discharge rate found in part (a) with the mass discharge rate expected for a liquid.

SOLUTION:

1. Write the dimensional functional relationship.

$$\dot{m} = f_1\left(D_E, g, \theta, \mu_{pp}, \mu_{pw}, \rho_b\right) \tag{1}$$

where $\dot{m}$ is the mass discharge rate from the hopper, D_E is the hopper exit diameter, g is the acceleration due to gravity, θ is the hopper wall angle, μ_{pp} and μ_{pw} are the friction coefficients between particles and between particles and the hopper walls, respectively, and ρ_b is the bulk density of the material at the hopper exit (the bulk density is the density of the particulate material including the void space between particles).

2. Determine the basic dimensions of each parameter.

$$[\dot{m}] = M/T$$
$$[D_E] = L$$
$$[g] = L/T^2$$
$$[\theta] = -$$
$$[\mu_{pp}] = [\mu_{pw}] = -$$
$$[\rho_b] = M/L^3$$

3. Determine the number of Π terms required to describe the functional relationship.

# of variables = 7 ($\dot{m}$, D_E, g, θ, μ_{pp}, μ_{pw}, ρ_b)
# of reference dimensions = 3 (L, T, M)

$$(\#\ \Pi \text{ terms}) = (\#\text{ of variables}) - (\#\text{ of reference dimensions}) = 7 - 3 = 4 \tag{2}$$

4. Choose three repeating variables by which all other variables will be normalized (same # as the # of reference dimensions).

D_E, g, ρ_b

5. Make the remaining non-repeating variables dimensionless using the repeating variables.

$$\Pi_1 = \dot{m} D_E^a g^b \rho_b^c \tag{3}$$

$$\Rightarrow\ M^0 L^0 T^0 = \left(\frac{M}{T}\right)\left(\frac{L}{1}\right)^a \left(\frac{L}{T^2}\right)^b \left(\frac{M}{L^3}\right)^c \tag{4}$$

$$\begin{aligned} M&: \quad 0 = 1 + c & & a = -\tfrac{5}{2} \\ L&: \quad 0 = a + b - 3c \quad \Rightarrow & & b = \tfrac{1}{2} \\ T&: \quad 0 = -1 - 2b & & c = -1 \end{aligned} \tag{5}$$

$$\therefore \Pi_1 = \frac{\dot{m}}{\rho_b g^{1/2} D_E^{5/2}} \tag{6}$$

$\Pi_2 = \theta$ (angles are dimensionless)

$\Pi_3 = \mu_{pp}$ (friction coefficients are dimensionless)

$\Pi_4 = \mu_{pw}$ (friction coefficients are dimensionless)

6. Verify that each Π term is, in fact, dimensionless.

$$[\Pi_1] = \left[\frac{\dot{m}}{\rho_b g^{1/2} D_E^{5/2}}\right] = \frac{M/T}{\left(M/L^3\right)\left(L^{1/2}/T\right)\left(L^{5/2}\right)} = 1 \quad \text{OK!}$$

$$[\Pi_2] = [\theta] = 1 \quad \text{OK!}$$

$$[\Pi_3] = [\mu_{pp}] = 1 \quad \text{OK!}$$

$$[\Pi_4] = [\mu_{pw}] = 1 \quad \text{OK!}$$

7. Re-write the original relationship in dimensionless terms.

$$\boxed{\frac{\dot{m}}{\rho_b g^{1/2} D_E^{5/2}} = f_2\left(\theta, \mu_{pp}, \mu_{pw}\right)} \tag{7}$$

If the wall angle and frictional properties remain constant, then doubling the exit diameter increases the mass flow rate by a factor of $2^{5/2} \approx 5.66$.

The mass discharge rate for a liquid discharging from the hopper is given by:

$$\dot{m} = \rho V_E \frac{\pi D_E^2}{4} \tag{8}$$

where ρ is the liquid density and V_E is the liquid speed at the hopper exit. The liquid speed may be found using Bernoulli's equation applied along a streamline from the hopper free surface, located a height, H, above the hopper exit, to the hopper exit. On both surfaces the fluid pressure is atmospheric and, hence:

$$V_E = \sqrt{2gH} \tag{9}$$

assuming that the kinetic energy of the upper free surface is negligible (i.e., it moves at a small velocity). Comparing Eqs. (7) and (8) shows that the mass discharge rate for a liquid depends upon the height of liquid above it while for a particulate material the discharge rate is independent of material height. In addition, the discharge rate for a particulate material is more sensitive to the hopper exit diameter (varying with $D_E^{5/2}$) than it is for a liquid (varying with D_E^2).

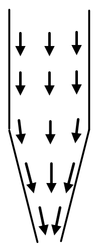
Notes:

1. Beverloo et al. (1961) observed that the experimental data for mass discharge rate from a flat-bottomed hopper is better fit using the following relation:

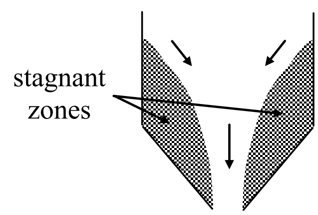
$$W = c\rho_b g^{1/2}\left(D_E - kd\right)^{5/2} \qquad \textbf{Beverloo Mass Flow Rate Correlation} \qquad (10)$$

where c is a constant incorporating the hopper wall angle and frictional properties (function f_2 in Eq. (7)), k is a constant that depends on the geometry of the exit and particle shape, and d is the effective diameter of the particles. The factor kd accounts for the fact that there is an annular zone at the periphery of the exit within which there are few particles. Hence, the effective exit diameter is reduced. The parameter k typically varies between 1.3 – 2.9 with a value of $k \approx 1.5$ for spherical particles. Angular particles have somewhat larger values for k. A value of $k \approx 1.4$ is a good general estimate if no discharge rate test data is available.

Beverloo et al. also observed that for funnel flow hoppers the parameter c is nearly independent of the friction coefficients, μ_{pp} and μ_{pw}, and the hopper wall angle, θ, and remains at a constant value of $c \approx 0.58$. A funnel flow hopper is one in which material remains stagnant adjacent to the hopper walls. A mass flow hopper is one in which all of the material flows simultaneously within the hopper.



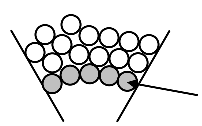
A mass-flow hopper.



A funnel-flow hopper.

3. The bulk density, ρ_b, in Eqs. (7) and (10) is <u>not</u> the bulk density of the material within the hopper. Studies have shown that the discharge rate from a hopper is independent of how the material is originally filled into the hopper. Instead, ρ_b is the bulk density of the *flowing* material. Since we often don't know the flowing bulk density of the material a priori, one can use the bulk density measured by loosely filling a container. The resulting predicted mass flow rate is typically within 5% of the measured value.

4. Blocking of the hopper exit can occur when the exit diameter is less than about six times the particle diameter. When the exit is smaller than this value, particles can form a mechanical arch that can support the force exerted by the material above it.



References:

Beverloo, W.A., Leniger, H.A., and Van de Velde, J., 1961, "The flow of granular solids through orifices," *Chemical Engineering Science*, Vol. 15, p. 260.