A model test of a tractor-trailer rig is performed in a wind tunnel. The drag force, F_D, is found to depend on the frontal area, A, wind speed, V, air density, ρ, and air viscosity, μ. The model scale is 1:4 (e.g., 1 m in the model is equivalent to 4 m in the prototype), frontal area of the model is $A = 0.625\ m^2$.

a. Obtain a set of dimensionless parameters suitable to characterize the model test results.
b. If the drag force on the full-scale vehicle traveling at 22.4 m/s is to be predicted from model testing, what should be the wind tunnel air speed? Assume that the air conditions are the same for the model and prototype.
c. When tested at the wind speed found in part (b), the measured drag force on the model was $F_D = 2.46$ kN. Estimate the aerodynamic drag force on the full-scale vehicle.
d. Calculate the power needed to overcome the full-scale drag force.

SOLUTION:

1. Write the dimensional functional relationship.

$$F_D = f_1\left(A, V, \rho, \mu\right)$$

2. Determine the basic dimensions of each parameter.

$$[F_D] = \frac{ML}{T^2}$$
$$[A] = L^2$$
$$[V] = \frac{L}{T}$$
$$[\rho] = \frac{M}{L^3}$$
$$[\mu] = \frac{M}{LT}$$

3. Determine the number of Π terms required to describe the functional relationship.

   # of variables = 5 (F_D, A, V, ρ, μ)
   # of reference dimensions = 3 (L, T, M)

   (# Π terms) = (# of variables) – (# of reference dimensions) = 5 – 3 = 2

4. Choose three repeating variables by which all other variables will be normalized (same # as the # of reference dimensions).

   A, V, ρ

5. Make the remaining non-repeating variables dimensionless using the repeating variables.

$$\Pi_1 = F_D A^a V^b \rho^c$$

$$\Rightarrow \; M^0 L^0 T^0 = \left(\frac{ML}{T^2}\right)\left(\frac{L^2}{1}\right)^a \left(\frac{L}{T}\right)^b \left(\frac{M}{L^3}\right)^c$$

$$\begin{aligned} M&: \quad 0 = 1 + c & & a = -1 \\ L&: \quad 0 = 1 + 2a + b - 3c & \Rightarrow \quad & b = -2 \\ T&: \quad 0 = -2 - b & & c = -1 \end{aligned}$$

$$\therefore \Pi_1 = \frac{F_D}{\rho V^2 A}$$ (This is a drag coefficient!)

$$\Pi_2 = \mu A^a V^b \rho^c$$

$$\Rightarrow \; M^0 L^0 T^0 = \left(\frac{M}{LT}\right)\left(\frac{L^2}{1}\right)^a \left(\frac{L}{T}\right)^b \left(\frac{M}{L^3}\right)^c$$

$$\begin{aligned} M&: \quad 0 = 1 + c & & a = -\tfrac{1}{2} \\ L&: \quad 0 = -1 + 2a + b - 3c & \Rightarrow \quad & b = -1 \\ T&: \quad 0 = -1 - b & & c = -1 \end{aligned}$$

$$\therefore \Pi_2 = \frac{\mu}{\rho V \sqrt{A}} \quad \text{or} \quad \Pi_2 = \frac{\rho V \sqrt{A}}{\mu}$$ (This is a Reynolds number!)

6. Verify that each Π term is, in fact, dimensionless.

$$[\Pi_1] = \left[\frac{F_D}{\rho V^2 A}\right] = \frac{ML}{T^2}\frac{L^3}{M}\frac{T^2}{L^2}\frac{1}{L^2} = 1 \quad \text{OK!}$$

$$[\Pi_2] = \left[\frac{\rho V\sqrt{A}}{\mu}\right] = \frac{M}{L^3}\frac{L}{T}\frac{L}{1}\frac{LT}{M} = 1 \quad \text{OK!}$$

7. Re-write the original relationship in dimensionless terms.

$$\boxed{\frac{F_D}{\rho V^2 A} = f_2\left(\frac{\rho V\sqrt{A}}{\mu}\right)} \tag{1}$$

To maintain similarity, the dimensionless terms must be the same between the model and prototype,

$$\boxed{\left(\frac{\rho V\sqrt{A}}{\mu}\right)_M = \left(\frac{\rho V\sqrt{A}}{\mu}\right)_P} \Rightarrow \boxed{\left(\frac{F_D}{\rho V^2 A}\right)_M = \left(\frac{F_D}{\rho V^2 A}\right)_P} \tag{2}$$

To determine the model testing wind speed, keep the Reynolds numbers the same between scales (the Pi term on the right hand side of Eq. (1)),

$$\left(\frac{\rho V\sqrt{A}}{\mu}\right)_M = \left(\frac{\rho V\sqrt{A}}{\mu}\right)_P, \tag{3}$$

$$V_M = V_P\underbrace{\left(\frac{\rho_P}{\rho_M}\right)}_{=1}\underbrace{\left(\frac{\mu_M}{\mu_P}\right)}_{=1}\underbrace{\left(\frac{A_P}{A_M}\right)^{1/2}}_{=(16/1)^{1/2}} \tag{4}$$

(same air properties; $L_P/L_M = 4/1 \Rightarrow A_P/A_M = (4/1)^2 = 16/1$)

$$\Rightarrow \boxed{V_M = 4V_P = 89.6\text{ m/s}}. \tag{5}$$

The force on the prototype is found using the other Pi term,

$$\left(\frac{F_D}{\rho V^2 A}\right)_M = \left(\frac{F_D}{\rho V^2 A}\right)_P$$

$$F_D\big|_P = \underbrace{F_D\big|_M}_{=2.46\text{ kN}}\underbrace{\left(\frac{\rho_P}{\rho_M}\right)}_{=1}\underbrace{\left(\frac{V_P}{V_M}\right)^2}_{=\left(\frac{22.4\text{ m/s}}{89.6\text{ m/s}}\right)^2}\underbrace{\left(\frac{A_P}{A_M}\right)}_{=\left(\frac{16}{1}\right)} \tag{6}$$

(Note the same air is used in both the model and prototype.)

$$\boxed{\therefore F_D\big|_P = 2.46\text{ kN}}$$

The power required to overcome the prototype drag force is:

$$P_P = \underbrace{F_{D_P}}_{=2.46\text{ kN}} \cdot \underbrace{V_P}_{=22.4\text{ m/s}} \tag{7}$$

$$\boxed{\therefore P_P = 55.1\text{ kW}}$$